Ollins Marshall M.D.
1885.

THE

MECHANISM

OF THE

OSSICLES OF THE EAR

AND

MEMBRANA TYMPANI.

BY

H. HELMHOLTZ,

PROFESSOR OF PHYSIOLOGY IN THE UNIVERSITY OF BERLIN, PRUSSIA.

TRANSLATED FROM THE GERMAN, WITH THE AUTHOR'S PERMISSION,

BY

ALBERT H. BUCK AND NORMAND SMITH,

OF NEW-YORK.

WITH TWELVE ILLUSTRATIONS.

NEW-YORK:

WILLIAM WOOD & CO., 27 GREAT JONES STREET.

1873.

NOTICE.

Prof. Helmholtz's essay on The Mechanism of the Ossicles of the Ear and Membrana Tympani, originally published in the 1st volume of *Pflüger's Archiv für Physiologie, Bonn*, 1869, is the only treatise in any language which enters fully into the anatomical, physiological, and mathematical aspects of the question, and will undoubtedly remain for many years to come the authoritative treatise on this subject.

In view of the great importance of this essay to those interested in the department of otology, the undersigned have attempted to translate it into English. The style of writing of the distinguished physiologist is so exceedingly condensed that some allowance will be made, we trust, for the evident lack of smoothness in the English version.

A. H. B. and N. S.

CONTENTS.

MECHANISM

OF THE

OSSICLES OF THE EAR.

From a notice found among the papers of the late B. Riemann, and recently published in the *Zeitung für rationelle Medicin*, we learn what views this man of unusual penetration—alas! too soon lost to science—took, during the last months of his life, regarding the problems of physiological acoustics, and why so few of them had thus far met with a solution. And here, too, we find that he had discovered the true source of all difficulty, and the one toward which all scientific efforts must henceforth be directed. He proposes, as the chief task of aural mechanics, to explain how the apparatus of the middle ear can transmit from the air to the fluid of the labyrinth such extraordinarily fine shades of vibration—as we know it actually does. He proves, by calculation, that the excursions of the stirrup, in the fainter (though yet clearly to be distinguished) tones, must be so small as to escape detection, even with the highest powers of our modern microscopes. To transmit regularly and accurately vibrations of such delicacy, he holds that there must be a corresponding accuracy and precision in the vibrations of the transmitting apparatus.

At the same time, he says he will be obliged to oppose in many particulars the theory of the mechanism of hearing as developed by me in the *Lehre der Tonempfindungen*. In this connection, I must remark that I myself at the time consid-

ered the description of the vibrations of the apparatus of the middle ear, as given in chapter 1, section 6, of the work in question, to be simply preliminary, and gathered from foreign sources. It was impossible for me at the time to make any investigations of my own into this question, although I fully recognized the necessity for new investigations. In the description which I gave in the same work I adopted, in its most essential features, the theory of Edward Weber,* which, compared with former theories, is a decided advance. It is, in the main, correct, although wanting in certain details which are indispensable to its completeness. It struck me that the chief difficulty in this theory lay in the existence of a joint between the hammer and anvil. According to Weber's description, the hammer and anvil constitute an immovable angular lever, whose axis of rotation is drawn through the processus Folianus of the hammer and the end of the processus brevis of the anvil. But how was the existence of a joint, surrounded by a weak and loose capsular membrane, allowing motion in all directions, possible in the midst of a lever whose vibrations must needs be of the greatest fineness and accuracy?

As soon as the completion of my work on physiological optics afforded me time for other investigations, I took the above question into consideration and had obtained nearly all of the following results before seeing Riemann's Notizen.† The solution of the difficulties was obtained by a closer investigation into the mechanics of the joints and attachments of the bones of the ear, and proved, in fact, to be entirely different from the one proposed by the celebrated mathematician. Besides, I must oppose his statement "that it is the task of the apparatus of the middle ear to transmit to the fluid of the labyrinth the changes in atmospheric pressure at every moment of time, with perfect accuracy and constant relative strength," because I consider this in nowise proven by the facts of the case. Accuracy in perception requires only that every tone of a

* Berichte über die Verhandlungen der Königl. Sächs. Ges. d. Wissenschaften zu Leipzig. Math. Phys. Klasse. 1851, Mai 18, S. 29–31.

† Short notice of them in the Heidelberger Jahrbücher. July 26th and August 9th, 1867.

given pitch should cause the same sensation, both in kind and intensity, every time that it is reproduced. It is a well-known fact that tones of a certain pitch produce an uncommonly strong impression upon the ear. We shall mention further on other new examples of abnormities.

§ 1.

Results due to the small Dimensions of the Auditory Apparatus.

The most important step in advance made by Edward Weber in the theory of the transmission of sound in the ear—a step which has received much less consideration than it deserves—seems to me to be the view that, in the transmission of sound-vibrations, the bones of the ear and the petrous portion of the temporal bone are to be considered as solid, incompressible bodies, and the fluid of the labyrinth as an incompressible fluid. He rightly declares that in the case of these bodies and fluids there can be no question as to the transmission of waves of condensation and rarefaction, but that the bones of the ear must be considered as solid levers, and the fluid of the labyrinth as a mass only to be moved as a whole.

I shall take the liberty of going more minutely into this special topic, inasmuch as it forms the basis of the subsequent investigations.

If in an elastic medium, be it a solid, fluid, or gas, whose three dimensions are infinitely extended, there be produced plane waves answering to a simple tone, these will pass through the elastic mass with the rapidity which belongs to that given tone, and produce at different points of the mass either displacement of the ultimate particles, or even condensation, where it is caused by longitudinal vibrations. If at a given point of the mass there are particles in a state of extreme displacement upward, at the same moment of time, there will be, at a distance of half a wave's length, particles in a state of extreme displacement downward; and the same is true of all other directions of displacement. Between these upper and lower limits of extreme displacement—which must be at least half a wave's length apart, as we have seen—we shall find in a continuous

line of transition the lesser degrees of displacement upward, the zero point of this displacement, and the lesser degrees of displacement downward, so that *the difference in displacement of two oscillating particles, whose distance from one another is infinitely small compared with the wave-length, is itself infinitely small compared with the entire amplitude of displacement.* If we limit ourselves in such a case to the consideration of a small portion of the vibrating mass, all of whose dimensions shall be infinitely small compared with the wave-length, then all the relative displacements of the individual points of this mass, among themselves, will be infinitely small compared with the amplitude of the entire vibrations, which in their turn must be considered as infinitely small compared with the wave-length, where sound vibrations are regularly produced. These relative displacements of the individual particles of the small mass (which we imagine to be taken out from the whole) among themselves are, therefore, infinitely small magnitudes of second order compared with the wave-length, and infinitely small magnitudes of first order compared with the amplitudes of vibration, and with the linear dimensions of the small mass to which they belong: that is to say, the small mass acts in the present instance just as an absolutely immovable body would.

The conditions remain the same when a large number of plane waves, belonging to the same simple tone, pass through the elastic mass; and also when spherical waves spread themselves through it, taking their start from any centre whatsoever of excitement within the mass, excepting, however, in the immediate neighborhood of punctiform or linear centres of excitement, whose appearance, however, is more a mathematical fiction than a practical reality.

The same law applies also to solid elastic bodies, provided their substance is not infinitely extended in all directions, but has limits against which the waves of sound may strike and be thrown back toward the centre of the mass. It is here, however, presupposed that either no linear dimension of the vibrating mass shall be very small, compared with the wave-length, or that this should be the case with all the dimensions of the vibrating mass at the same time, so that no one of them should be very

small compared with the others, as is the case, for instance, in disks, membranes, rods, and strings.

The proof of these laws is easily deduced from the well-known laws respecting the form and mode of vibration of plane waves —so long, of course, as there is only question of plane waves of simple tones in masses of infinite extent. On the other hand, the influence of boundary-planes (Grenzflächen) and of the last-named conditions has been elucidated by Kirchoff, in his treatise on the equilibrium and vibration of an infinitely thin elastic rod.* In this treatise, it is true, only the equilibrium of such elastic masses is taken into consideration, and it is there proven that forces, which are infinitely small compared with the constant of elasticity of the body, and which are brought to bear partly on the central and partly on the superficial portion of the elastic mass, cause only infinitely small relative displacements of such particles as lie within finite distance of each other, so that the differential quotients of the displacements, as taken from the co-ordinates, also remain finite. On this last point the question chiefly hinges. For, if these differential quotients are finite magnitudes, then, in masses of infinitely small linear dimensions, the relative displacements of the individual particles are infinitely small, compared with the total absolute displacements which such masses experience.

The above-mentioned law, which Kirchoff has demonstrated for the condition of equilibrium, the forces involved being infinitely small, may also, by means of d'Alembert's rule, be applied to the condition of motion, provided the accelerations which the particles of the mass experience during motion are considered as the forces which disturb the elastic body. These now, when they belong to vibrations whose amplitude, compared with the length of the wave, is infinitely small, are themselves infinitely small, and answer, therefore, to Kirchoff's acceptation of infinitely small † disturbing forces.

* Borchardt's Journal für reine und angewandte Mathematik LVI., in § 1 of the treatise in question.

† If A be the amplitude of vibration, n the number of vibrations of a simple tone, t the time, and c a constant determining the phasis, then has s, the variable departure from the position of equilibrium, the following value:

$$S = A \sin \left\{ 2 \pi n t + c \right\}$$

The law demonstrated by Kirchoff, and applied to the present case, might be thus expressed :

In immovable elastic bodies, all of whose linear dimensions are not infinitely small compared with the wave-length, or at least none of which are infinitely small compared with the rest, *vibrations of a simple tone*, whose amplitude is infinitely small compared with the wave-length of the same kind of vibrations in masses of infinite limits, *produce upon two points of the elastic body*, whose distance from one another is infinitely small compared with the same wave-length, *relative displacements*, which are themselves infinitely small compared with the entire amplitude of the vibrations.

That is to say, then, that, under the restrictions mentioned, masses whose linear dimensions are all small compared with the wave-length act exactly like absolutely solid bodies ; or, that the changes in form which they undergo can be disregarded when compared with the entire amplitude of their vibrations.

If we now take into account that in air the wave-lengths of the tones constituting our musical scale—that is, from C_{-1} with 33 vibrations to c_5 with 4224 vibrations—vary from 8 to 1000 cm. ; that in water the same waves are more than 4 times, in brass about 11 times, in copper 12 times, in steel and glass more than 15 times greater than in air; that, on the other hand, the dimensions of the bones of the ear and of the labyrinth are only small fractions of a centimetre, the important

If μ represent the volume of the small part, then k, the power used to accelerate the same, is equal to

$$k = \mu \frac{d^2 s}{dt^2} = -4\,\pi^2\,n^2\,\mathrm{A}\sin\left\{2\,\pi n t + c\right\}$$

If now λ be the wave-length, and a the rate of progress for this kind of vibrations in masses of unlimited extent, then

$$n = \frac{a}{\lambda};$$

and for the maximum of k, which appears as often as the sinus of the formula given for it equals $\pm$ 1 :

$$\frac{k}{a^2} = \pm\,4\,\pi^2\,\frac{\mathrm{A}}{\lambda^2}.$$

k is therefore infinitely small compared with a^2, provided A is infinitely small compared with λ ; and a^2 multiplied by the density is equal to the constant of the elastic resistance, which in this kind of comparison assumes a value.

conclusion follows that the dimensions of the elastic solid and fluid masses, constituting the organ of hearing are all at best only very small fractions of the wave-lengths of those tones which we commonly hear, and which our ear can readily appreciate.

We are, moreover, to conclude from what has already been said that, in the vibrations of the auditory apparatus, of the bones of the ear and of the petrous bone, caused by the tones ordinarily appreciable by the ear, the particles of each of these small masses undergo displacements among one another, which are infinitely small compared with the amplitude of the sound-vibrations producing them; that is to say, that they act very nearly like absolutely solid bodies.

The final reason for this peculiarity of motion is to be found in the very great rapidity with which the influence of every shock, communicated to one of these small solid masses, is transmitted through it. This rapidity is so great that the time required for transmission of the shock may, as a rule, be considered as infinitely small when compared with the duration of the individual sound-vibrations, and its action as instantaneously conveyed throughout the entire mass.

An incompressible fluid, inclosed within solid walls, differs from one that is compressible in the fact that here, too, every shock communicated to one part of its superficies is instantly transmitted through the entire fluid, and sets every portion of it instantaneously in motion; while in a compressible fluid one wave starts from its point of origin, runs its course with a certain speed, and sets alternately the different portions of the fluid in motion. If, therefore, in the case of the fluid of the labyrinth, the dimensions of the whole mass are infinitely small compared with the wave-length, and the walls of the petrous bone inclosing the fluid are strong enough to be considered as absolutely immovable beneath the small pressure exerted in this instance against them, then the transmission of the shock throughout the entire mass is practically instantaneous, and the fluid of the labyrinth may be said to act under the influence of vibrations of sound precisely as a fluid absolutely incompressible, and therefore incapable of transmitting the vibrations of sound, would do under the same circumstances.

Finally, it is necessary, at least for the deeper and middle tones of the scale, that there should be an equality of pressure between the air contained in the middle ear and that of the external auditory canal. In the case of very high tones, those corresponding to the highest octave of the piano, the length of the auditory canal is very nearly equal to a quarter of a wave's length, to which circumstance is due the occurrence of those phenomena of resonance described by me in the *Lehre von den Tonempfindungen*.* At all events, the diameter of the external auditory canal is too small to permit of different phases of pressure or of speed at different points of the membrana tympani at the same moment, and we can, therefore, without hesitation consider the pressure as equal at all times over all parts of the membrane. This circumstance is likewise of great importance in the mechanism of the ear, for it excludes all possibility of one part of the membrana tympani being excited, while the rest is not; the part excited being dependent on the locality of the sound-giving body. Hence we have no other means of localizing sound except by noting the different degrees of intensity obtained by changing the position of the head and comparing the impressions made upon both ears.

The above-mentioned rule applies, as already stated, to bodies none of whose linear dimensions are infinitely small compared with the rest, consequently not to strings, membranes, rods, and disks. It is also liable to exceptions, as where the middle portion of the body in question is contracted and very narrow. Among the component parts of the auditory apparatus, the membrana tympani is the only one which falls under the head of exceptions. In point of fact, those bodies which are very thin at one spot, or in one direction, are capable of performing comparatively slow vibrations; for, owing to their slight thickness, they offer but a feeble elastic resistance, return slowly to a state of equilibrium, and vibrate at a much slower rate than is the case with oscillations in thick masses of the same nature.

That the bones of the ear do not come under the head of exceptions, is easily shown by comparing them with the metallic rods or tongues which are used in the production of high tones.

* Pages 175, 176.

The tongues employed to produce the highest tones of the musical scale in a harmonium are relatively very long and thin when compared with the dimensions of the bones of the ear, and no one, who has any experience in the tones which belong to, or can be produced by such solid bodies, could for a moment doubt that, were it possible to put into regular vibration such small masses as the bones of the ear, including the relatively slender stirrup, these would give forth tones of such enormous height that to our ear they would probably no longer be perceptible—tones lying far beyond the limits of our musical scale.

The relation sustained by the bones of the ear to the vibrations of sound is practically the same as in an iron rod, when hung up and caused to vibrate as a pendulum. Such a rod is elastic and yielding, and is capable of several kinds of vibration ; but these vibrations take place at the rate of several hundred per second, while as a pendulum it swings perhaps only once in a second. If such a pendulum is caused to vibrate by a force exercised periodically, the periods amounting to one or more seconds, or to larger fractions of a second, each blow communicated by this power to one point of the rod can traverse the same hither and thither several hundred times before the blow belonging to the next period is given, and thus the effect of the blow can be transmitted thoroughly to every part of the mass before even a small fraction of the period of a vibration has passed. Under these circumstances the pendulum vibrates practically as an absolutely solid body, that is, its real motion is not to be distinguished from the motion of such a body, not even by means of the most delicate methods of observation. Entirely different is the action of the pendulum when we cause it to vibrate by means of a tone whose pitch approximates that of the rod. Then it vibrates, no longer according to the laws of a pendulum, but as a vibrating elastic rod.

The same is true of the bones of the ear. As long as the periods of vibration of the tones which these must transmit are very great compared with those of the bones themselves, so long will the latter act, practically, as absolutely solid bodies.

§ 2.

Anatomy of the Membrana Tympani.

Before passing on to the discussion of the mechanism of the apparatus of the middle ear, I must make one or two anatomical remarks, not with the view of bringing forward any thing materially new, but simply to give prominence to a number of small points, which as a rule are merely noticed and then passed over by the anatomist, but which gain importance in a more thorough investigation of their physiological bearings.

The opening in which the membrana tympani is set is formed from the squamous portion of the temporal bone and from what was once the annulus tympanicus, both of which in adults are firmly connected by a bony union; not so firmly, however, but that in chiseling out a preparation of the ear, a break is likely to occur at this very spot of union—a circumstance which I found very annoying in exposing to view the connections of the upper part of the membrana tympani. Even on the dried adult bone this line of separation is still pretty clearly marked by two prominent bony spurs, which rise up, before and behind, on the boundary between both parts; these separate a lower part,

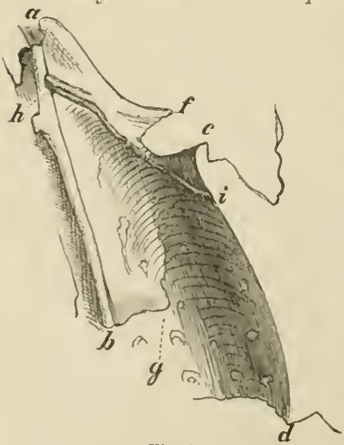

Fig. 1.

which is nearly oval in form and contains a rim for the attachment of the membrana tympani, from an upper part, which is irregular in outline and more strongly concave. The former belongs to the os tympanicum, the latter to the os squamosum. Fig. 1 represents the upper and anterior wall of the bony portion of the external auditory canal. The line of section was drawn parallel to this wall. *a b* is the surface of section of the anterior wall, which separates the auditory canal from the joint of the jaw; *c d* is the line of section through the posterior wall; *b d* is the outer opening of the auditory canal; a slight furrow *h i*, which in the

engraving is more strongly marked than is actually the case in nature, represents the line of attachment of the membrana tympani. Traces of the fissure, which in the fœtus divides the anterior upper border of the annulus tympanicus from the squamous portion, may still be seen running from the point *f* in the direction of *g*. Between *a* and *h* the same fissure (fissura Glaseri) is recognizable. The projecting point at *f*, which plays an important part in the attachment of the hammer, is called by Henle the spina tympanica posterior, in contradistinction to another more distinctly marked point in the fœtus, on the anterior end of the annulus, at its outer anterior angle, which he calls the spina tympanica anterior, and which, on the much broader os tympanicum of the adult, answers to the point *g*. The latter, however, lies flat upon the corresponding surface of the squamous bone and no longer stands out as a spur. On the posterior end of the above-mentioned recess, and corresponding to a point in Fig. 1 between *c* and *i*, there can be seen a blunt and less prominent projection of the rim, in which the membrana tympani is inserted, which we shall frequently have occasion to speak of in describing its attachments. In order to avoid errors which might arise through my giving Henle's name of spina tympanica posterior to the anterior point *f*, I shall take the liberty of applying to it the name of spina tympanica major, and to the posterior point at *i* that of spina tympanica minor.

The neck of the hammer fits into the recess lying between *f* and *c* in such a manner that the point at *f* almost touches it. The line of attachment of the membrana tympani also shows a slight and ill-defined depression where it passes near the points *f* and *i*. And just here, moreover, the line is less sharply defined than lower down on the part formed from the os tympanicum; and here, too, slight pressure with a blunt instrument will loosen the membrana tympani from its attachments. In fact, it is more truly attached to the cutis than to the bone.

This recess in the upper border we shall call the Rivinian recess, as it includes the opening described by Rivini, an opening which represents the last trace of the original visceral cleft, but which in the majority of normal adults does not exist.

Although normally no opening exists there, still the Rivinian

recess is filled with a loose part of the drum-head, which, as it appears beneath the thin cutis, is seen to consist of bundles of loosely interwoven connective tissue, which give passage to vessels and nerves and are easily separated. (Membrana flaccida Shrapnell.) For this reason, abscesses are wont to perforate at this point, and here too, in making preparations of the cutis layer, artificial openings are readily made. The difference in tension and consistency between this upper part of the drum-head and the rest of the membrane can easily be felt by passing the blunt end of a sewing-needle over the surface of the membrane, in a preparation where the attachments of the bones and of the membrana tympani are still undisturbed. It is then readily perceived that between the spina tympanica major and minor there is a pretty tense cord of fibres, into which the processus brevis of the hammer is inserted in a direction toward the anterior border. This cord forms the upper border of the lower and firmer part of the membrane. As soon as the exploring needle passes beyond it, it sinks suddenly into the Rivinian recess, while pressing before it the loose cutis and mass of connective tissue. And if, moreover, we examine carefully the vaulting of the outer side of the membrana tympani, in a suitable preparation and with oblique light, we can generally make out this cord running from the processus brevis mallei toward the spina tympanica minor. As far as I could ascertain, this cord is formed from the peculiar tendinous fibres of the membrana tympani. We shall call it the upper cord of attachment of the membrana tympani. It forms the boundary for that part of the membrane which has to be taken into consideration in vibrations of sound.

On the inner side, the membrana flaccida is continued on from its line of insertion into the tissue of the fold of mucous membrane which forms what has been described by Tröltsch as the posterior pocket of the membrana tympani, and in whose lower free border lies the chorda tympani. The line of insertion of the membrana tympani unites with that of the above-mentioned fold at the bottom of the Rivinian recess; here their attachment to one another is stronger than to the bone; posteriorly, however, the line of insertion of the fold of mucous membrane does

not run parallel with that of the membrana tympani, but pursues its course along the sharp border of the wedge-shaped bony process represented by *c* in Fig. 1. The outer surface of this process lies parallel with the membrana tympani and a short distance to the inside of it, and can even be seen from the outside as a whitish object shining through the semi-transparent membrane. Lower down on the border of this process is the opening which gives egress to the chorda tympani. The smaller recess, which can be seen behind the sharp border near *c* in Fig. 1, represents a section of the funnel-shaped projection of the canal of the chorda. The fold of mucous membrane forming the posterior pocket of the membrana tympani reaches down as far as the exit of the nerve, which itself forms the border of the pocket.

The line where the fold of mucous membrane comes in contact with the membrana tympani runs from the highest point of the Rivinian recess forward toward the processus brevis of the hammer. This portion of the fold separates the smaller anterior from the larger posterior pocket. Its line of attachment on the hammer we shall describe hereafter.

The Rivinian recess lies above and a little in front of the membrana tympani. Its greatest diameter extends in a nearly perpendicular line downward from the posterior end of the recess, above the spina tympanica minor. I have measured its length in a number of specimens, and find it agrees with that given by Tröltsch—9 to 10 mm. The smallest diameter is in a nearly horizontal direction, and begins somewhat under the spina tympanica major. Its length I found to be from 7½ to 9 mm. These measurements are, as a general thing, the same in infantile skulls as in those of adults.

As is well known, the inner end of the external auditory canal is pointed inward and a little downward; and, besides, the plane that passes through the groove in which the membrana tympani is inserted is inclined at an angle of 55° to the axis of the external auditory canal, while the membranes of both sides form with each other an obtuse angle, opened upward, of 130° to 135°.

The membrana tympani is not stretched out flat in the ring

to which it is attached, but its centre or navel is strongly drawn inward by the handle of the hammer, with which it is united; for this reason the membrane has the shape of a funnel whose point or end corresponds to the tip of the handle of the hammer, and whose meridian lines are convexed toward the hollow of the funnel. In order to represent this form of the membrana tympani, a point of great importance in the mechanics of the conduction of sound, I took a cast with stearin of the upper wall of the external auditory canal and of the outer surface of the membrane, after having first removed the lower wall of the canal, without, however, disturbing any of the connections of the membrana tympani. Its outlines are represented in Fig. 2, just as I copied them in the camera clara; *a b* is the upper wall of the external auditory canal, *b c* the vertical outline of the membrana tympani.

Fig. 2.

From the figure it is very clear that the radii drawn on the surface of the membrana tympani are convexed outward toward the external auditory canal. At the same time, it can be seen that, as a result of this drawing in of the navel, the upper half of the membrane is made to lie in almost the same direction as the upper wall of the canal, while the lower half stands almost at a right angle with the axis of this canal. This last circumstance is of importance in the examination of the ear with the reflector, inasmuch as this perpendicular portion of the membrana tympani, which is situated, as a rule, just below the tip of the manubrium, reflects back through the external auditory passage the light that is thrown in upon it, and thus gives rise to the triangular "bright spot."

The outer surface of the membrana tympani, which is covered with an epithelial layer, the continuation of the horny epidermis of the skin of the external auditory passage, owes its property of reflecting light to the fat which it contains. In a very fresh specimen of the ear, drops of water can be seen running off from this fatty surface as from oiled paper.

The convexity of the meridians of the membrana tympani is least at that meridian in which the handle of the hammer lies.

The outline of the stearin cast answering to this part is represented in Fig. 3, the position of the hammer being marked by dotted lines. At the same time, it can be seen in this drawing that the navel lies somewhat under the true centre of the membrane.

Fig. 3.

The meridian in which the handle of the hammer lies extends upward and forward from the navel toward the anterior limit of the Rivinian recess, so that the processus brevis of the hammer, which forms the upper limit of the handle, comes to lie nearly back of the spur which is situated on the outer side of the line of attachment of the membrana tympani, and which answers to the inwardly pointing spina tympanica major. To this the hammer is attached partly by means of a compact ligament (ligamentum mallei anterius,) and partly by its so-called long process, (processus Folianus.) The latter, so long as it exists, lies in a furrow on the inner border of the process.

While, on the one hand, the tip of the manubrium draws the navel of the membrana tympani inward, on the other, the processus brevis at the base of the manubrium tends somewhat to press it outward.

The membrana tympani consists essentially of a peculiar tendinous membrane, which, although only one-twentieth of a millimetre thick, is yet comparatively very strong. Externally it is clothed with a thin continuation of the skin of the external auditory canal, internally by a thin continuation of the mucous membrane of the middle ear. Taken together, these layers have a thickness of 0.1 mm. The outer skin layer consists principally of a continuation of the epidermis, supported by a thin layer of loosely-woven bundles of connective tissue. It can be removed entire from the greater portion of the surface of the membrane, excepting at the Rivinian recess and along the handle of the hammer,* where it is more closely united with the thickened and cartilage-like tissue of the membrane. From the Rivinian recess along the upper wall of the external auditory canal there runs

* Gruber's obliquely descending fibres of the membrana tympani unite at this point with the fibres of the cutis, forming in a mechanical sense—although perhaps they must be separated histologically—the deepest layer of the same.

a line, along which the skin is more strongly attached to the bone. The fibres of the cutis dip down here into the fissura Glaseri, which at an earlier period was a cleft dividing the squamous portion from the os tympanicum, (Fig. 1, *fg.*)

The middle and stronger layer of the membrana tympani is fibrous, and consists partly of radiating, partly of circular fibres. The radiating fibres lie on the outer side, the circular on the inner side of the layer. In the anterior half of the membrane the radiating fibres proceed from the tip of the manubrium mallei as a centre. On the posterior half, however, they run nearly parallel to each other from the entire length of the manubrium. Their thickness is least along the border of the membrane and grows gradually thicker on approaching the end of the manubrium, where they are packed more closely together.

In the centre of the membrane the circular fibres form a very thin layer which gradually increases in thickness toward the periphery; at the extreme periphery, however, they disappear altogether (according to Gerlach), or at least (according to J. Gruber) form a very much thinner layer than in the centre. In the Rivinian recess the circular fibres are strongly developed and of a satin-like appearance; they form here a cord-like boundary for the upper side of the firmer part of the membrana tympani and intersect at a very small acute angle the radiating fibres, which at this point radiate, not from the navel, but from the processus brevis of the hammer. Here, too, they are intermingled with straggling fibres of the cutis.

The tendinous fibres of these layers are very dense and unyielding, they lie close to one another, and offer very great resistance to any distending force. Through their great power of elastic resistance they differ materially from the very much more yielding yellow elastic tissue. The substance of the membrana tympani swells in acetic acid and solutions of potash, as is the case with tendinous tissue, but not with the elastic tissue. I found that, like tendinous tissue, it would soon dissolve completely in a boiling potash solution, leaving behind mere traces of elastic tissue, consisting partly of vessels, and partly of a very thin continuous membrane—probably the base-

ment membrane of the mucous layer on the inner side of the membrana tympani.

This peculiarity of construction of the membrana tympani is a very important element in the mechanical working of this membrane, as we shall see further on. It is not to be considered as an elastic, yielding membrane, but as an almost inextensible one. Its want of capacity for yielding can be appreciated when one tries to tear it with needles, either after it has been removed and spread out upon a glass slide, or while it still remains attached in its natural position. It cannot be drawn out like a piece of rubber, or a softened animal bladder, but offers very powerful resistance to tension, and forms folds about the spot that is being stretched, as in a collodion membrane.

§ 3.

Attachments of the Hammer.

The manner in which the hammer is attached to the membrana tympani has been thoroughly described by J. Gruber in a monograph published recently by him. The part of the membrana tympani corresponding to the attachment of the hammer is thickened, partly by strong fibres of the cutis layer extending from the Rivinian recess along the manubrium, partly by an accumulation of fibro-cartilaginous tissue. The periosteum of the hammer, along both surfaces of the manubrium, is continuous with the fibro-cartilaginous layer, whose borders are thus closely united to the hammer. Near the lower end of the manubrium the union between the bone and the thickened tissue of the membrana tympani is very close; near the processus brevis, however, a looser layer intervenes between the bone and the membrane, or there may even be a kind of incomplete joint-space, which is limited on both sides by the closer union between the periosteum of the hammer and the borders of the cartilaginous layer, together with the fibrous tissue of the membrana tympani.

The hammer by means of its handle draws the navel of the membrana tympani inward; to maintain a close union between these two parts the connection between them should be strongest at this point. At the processus brevis the hammer simply presses

against the membrana tympani; consequently here a less intimate union suffices, while at the same time it affords a possibility for slight motions of the hammer upon the membrane, a necessity whose conditions we shall investigate more thoroughly further on.

The second and relatively strongest attachment of the hammer is to the spina tympanica major. The end of this spur extends close up to the neck of the hammer, into the hollow at *d*, (Fig. 4,) just above the root of the processus Folianus. The hammer, in this sketch, is seen from the outside; *cp* is the head, *b* the processus brevis, *m* the handle, * the articular surface for the reception of the anvil. The processus Folianus lies along the inner border—that turned toward the tympanum—of the spina, (the margin *f a* of Fig. 1,) so that between *l*, the end of the processus Folianus, and the hollow at *d*, the border of the spina and the corresponding border of the processus Folianus run almost parallel to each other, with a space intervening between them of about $\frac{1}{3}$ mm. From the upper surface of thes pina a bony margin extends upward in such a manner as to fit pretty closely to the side of the hammer between *d* and *f*, the intervening space between the hammer and the bony margin being continuous with the space previously mentioned and of the same width with it. This entire fissure is bridged over with short and tense ligamentous fibres; longer fibres of the same kind start from the surface of the spina and from its downward projecting border, and, converging toward the point *d* of the hammer, envelop both the lower border and the outer surface of the processus Folianus, so that it lies completely hidden in this mass of tendinous fibres which forms the ligamentum anterius mallei and is covered by a fold of mucous membrane.

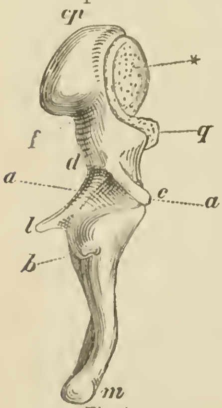

Fig. 4.

As for the processus Folianus of the hammer, I must remark that in children it is a long elastic blade of bone extending as far as to the fissura Glaseri. As regards, however, its condition in adults, I must side with those who describe it as having

dwindled down to a short stump. I would state that, in preparing a number of temporal bones, I took particular care to notice whether this process was not perhaps broken off in the attempt to loosen the hammer. To ascertain this, before the hammer had been at all disturbed in its natural position, I searched for the processus Folianus with the point of a fine needle, inserting it as a probe between the fibres of the ligamentum anterius mallei. In this way I could follow it distinctly for a short distance, when it came abruptly to an end in the midst of the anterior ligament, and I could feel nothing like a continuation of the bony blade, which would have been present if the process had simply been broken.

I must remark, moreover, that this remaining stump of the processus Folianus does not lie in immediate contact with the bony mass of the spina; it is attached to it throughout by means of short fibrous bands. Hence in a specimen where the hammer retains its natural position, and all of its attachments are preserved entire, it is possible, by pressing upon the base of the processus Folianus with the point of a needle, to move this part of the hammer not only upward and downward but also backward and forward, as far as the short fibrous bands of the ligamentum anterius will permit. The contact of bone with bone, if it existed here, would offer an obstacle to any one of these motions.

If we, therefore, leave out of the account the longer superficial fibres, the ligamentum anterius mallei appears in the main to be a very short and broad band whose line of insertion extends from l to f on the hammer (Fig. 4); from l to d it lies opposite the inner border of the spina tympanica major, but near the point d it is nearly opposite to a bony ridge which extends from d on the spina upward toward f. I must add, moreover, that, above and below, this band becomes lost in a fold of mucous membrane. Above, the fold of mucous membrane follows pretty closely the contour of the bone (see Fig. 4); it is here sickle-shaped and quite thin, because here the outer wall of the tympanum is everywhere in close proximity to the head of the hammer. This fold of mucous membrane finally terminates on the upper surface of the head

of the hammer, and in its border lies the short, round ligamentum mallei superius, which descends obliquely downward and outward upon the head of the hammer. Its office is, therefore, to restrain all motions of the latter outward.

The ligamentum anterius is prolonged downward from *l* into two folds of mucous membrane, one of which runs from the base of the processus Folianus toward the point *b* of the processus brevis. Its opposite line of insertion lies on the membrana tympani. This is the fold which separates the anterior from the posterior pocket of the membrana tympani, dividing them in such a way that the space above the processus brevis belongs chiefly to the posterior pocket.* The second prolongation of the ligamentum anterius downward is a thin fold with a free margin, which follows the lower border of *l*, (after having enveloped this process,) and then continues down along the contour of the bone as far as to the tendon of the tensor tympani muscle (Fig. 4). In the drawing this spot corresponds to where the curved line at *b* meets the contour line of the bone. This last-named fold is a limit between the anterior pocket and the cavity of the tympanum.

From the present group of ligaments and mucous folds, which in Fig. 4 follows always the contour line of the bone from *b* to *cp*, and at *d* is composed of the shortest and strongest fibres, a second group branches off at *d*, to which I shall give the name of ligamentum mallei externum. It springs from a rather prominent bony ridge on the hammer, which may be seen extending from *d* to *c* in Fig. 4, and is inserted into the sharp border of the Rivinian recess; at the same time it follows posteriorly the line of attachment of the posterior pocket of the membrana tympani (hence in Fig. 1, from *f* to *c* along the contour line of the drawing). This ligament consists of a number

*In the "Archiv für Ohrenheilkunde," vol. iii., pages 255–266, Dr. Prussack has given a description of the pockets of the membrana tympani, which differs from this. He describes the space above the processus brevis of the hammer as a special upper pocket, separated from the posterior pocket by a partition wall; I have never been able to find such an one. The supposed entrance into this pocket, above and in front of the top of the head of the hammer, leads into the space above the ligamentum mallei externum, and therefore not at all to the membrana tympani.

of distinct, satin-like, tendinous fibres, which radiate from the short crest of the hammer (lying between *d* and *c*) toward the much broader curved line of attachment on the temporal bone.

In Fig. 5, this ligament is represented as seen from above; *e g* is its line of attachment to the temporal bone. The tympanum in this preparation was opened from above, and its upper and outer wall sufficiently chiseled away to permit of a free view between this wall and the surface of the bones facing it.

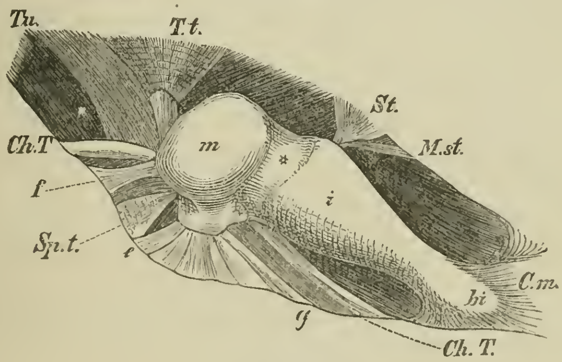

Fig. 5.

m is the head of the hammer, *i* the body of the anvil, *bi* the end of its short process, and *Tu* the entrance to the tube. Deep down a part of the stirrup *St* can be seen, and the tendon of its muscle *M.st*, and further on the tendon of the tensor tympani with the funnel-shaped osseous canal from which it issues. *Ch.T* is the chorda tympani, which marks the free border of the folds of mucous membrane limiting the pockets; at *f* are the upper fibres of the ligamentum mallei anterius, which arise above the spina tympanica major *Sp.t.* The prominent crest on the neck of the hammer, from which the fibres of the ligamentum externum radiate, is here distinctly visible.

The strongest and most tense bundle of fibres of this last-named ligament is the posterior one, which is inserted at *g*. The line of the direction which it follows, when continued, passes through the end of the spina. It is also this bundle which represents chiefly the axis of rotation of the hammer. I prefer, therefore, to call this posterior group of fibres of the

ligamentum externum by the special name of ligamentum mallei posticum, because in a mechanical sense it has indeed a special importance. In a specimen where all the attachments of the ossicles remain undisturbed, the great tension of these posterior fibres can be distinctly felt by pressing upon them with the point of a needle; at the same time the border of the fold of mucous membrane, in which the chorda tympani lies, will always be found in a relaxed condition; and moreover the anterior bundle of fibres of the ligamentum externum (at *e* in Fig. 5) is never very tense unless the tensor tympani is in a state of contraction, or the membrana tympani is being forced outward. By pressing harder with the needle upon the chord of the ligamentum posticum, the hammer can be made to incline appreciably. In forcing the membrana tympani inward or outward, it is this very group of fibres of the ligamentum externum which moves the least of all the attachments of the hammer. The reason for this slight displacement will appear further on.

If we suppose the line of direction of the ligamentum posticum continued on through the hammer, it will be found to meet and run in the same direction with the middle and strongest fibres of the ligamentum anterius which take their origin from the spina tympanica major. These two sets of fibres, which, although separated by the intervening body of the hammer, are still in a mechanical sense one band, we may call the axis-band of the hammer. This band alone is sufficient to hold the hammer in its natural position, even after the anvil has been carefully separated from it; but if the tendon of the tensor tympani be still tense, this position will then be really quite firm. In Fig. 4, the approximate position of the axis of the hammer is marked by a dotted line *a a*.

The cords forming the anterior portion of the ligamentum externum (Fig. 5, *e*) are made up of shorter fibres which are directed outward toward the edge of the membrana tympani, where it is attached at the bottom of the Rivinian recess. As they lie above the axis of the hammer, they oppose any movement of the manubrium mallei or of the membrana tympani outward toward the external auditory canal. Hence their es-

sential office is to restrain the rotation of the manubrium outward. In a suitable preparation, like that of Fig. 5, this fact can be readily verified. When the membrana tympani is pressed inward, or the head of the hammer outward, these fibres become relaxed. They allow only a slight rotation of the manubrium outward, even after the tendon of the tensor tympani, the stapedo-incudal joint, and the ligamentum superius mallei have been divided. By pressing upon these same fibres from above with the blunt end of a needle, and thereby putting them on the stretch, the inclination of the membrana tympani inward will be increased. Finally, it must be remarked that, whenever the tensor tympani is strongly stretched, the manubrium mallei is prevented from being drawn further inward by the tense condition of the membrana tympani, while the axis-band of the hammer is prevented from being pulled inward beyond a certain limit by the above-mentioned set of fibres of the ligamentum externum; when these become tense the limit has been reached. At this point, the traction of the tensor tympani will be transferred to these fibres, and can no longer affect the axis-band.

While the ligamentum externum, on the one hand, protects the axis-band of the hammer from being pulled too strongly inward, the upper and lower fibres of the ligamentum anterius, on the other hand, prevent it from being drawn too strongly upward or downward. If the hammer were to rotate about its attachment to the spina as a centre, with its head backward and the end of the manubrium forward, the upper fibres of the ligamentum anterius would be put upon the stretch; and if in the opposite direction, the lower fibres. Hence it happens that, even after the anvil has been removed, the ligaments hitherto described remain unaffected, the hammer is still able to resist any such inclinations, and remains pretty steady in its natural position. The uppermost fibres of the ligamentum anterius usually approach the head of the hammer in an inward direction, (as can be seen in Fig. 5 at *f*,) and hence, like the ligamenta superius and externum, they become tense when the membrana tympani is pushed outward.

The tension of these ligaments, in the natural state of things,

is increased by the elastic force of the comparatively strong musculus tensor tympani, whose tendon is attached to the hammer on the anterior half of its inner side facing the tube, at the commencement of the manubrium, and a little lower down than where the processus brevis projects from the side. In Fig. 9, the line of insertion of this tendon is shown extending from above obliquely downward and backward. The muscle, as is well known, lies in a special bony canal whose course runs parallel with and above the Eustachian tube, by means of which the cavity of the tympanum communicates with the pharynx. The further end of the muscle arises outside of this canal, from the under surface of the pyramidal portion of the petrous bone and from the cartilaginous portion of the Eustachian tube. It then proceeds through its appropriate canal, whose open end, toward the cavity of the tympanum, terminates in a spoon-like process: around this the tendon of the muscle passes, and then finally crosses the cavity of the tympanum obliquely (*T t*, Fig. 5) toward its point of insertion on the hammer. The direction followed by the tendon is nearly perpendicular to the plane drawn through the border of the membrana tympani, so that its line of traction varies only a little downward and forward from such a perpendicular. On the other hand, it forms a moderately acute angle with the lower portion of the handle of the hammer and with the anterior portion of its axis of rotation.

The tensor tympani is a penniform muscle; it originates from the periosteum of the upper wall of the canal in which it lies; its tendon lies next to the under surface of the canal, and presents a free, smooth surface to the smooth periosteum. The muscular fibres are rather short, and hence the tendon extends back to the very end of the canal. The periosteal tube which sheathes the muscle is continued over the tendon in its course through the cavity of the tympanum; its outer surface is there covered with the mucous membrane of that cavity. Toynbee calls this free part of the sheath the tensor ligament of the membrana tympani. The separation of the tendon from its sheath—if we compare the descriptions of different observers—seems to be more or less complete; in an anatomical collection of this city I have seen a specimen where the perfectly smooth

tendon was surrounded by a perfectly free sheath, just as Toynbee describes it; in microscopic sections, however, Henle has seen the two united by pretty strong bands of connective tissue. As the hammer, however, requires exceedingly little space for its excursions, there is no need whatever that the tendon should have much room for motion.

The tensor tympani draws the handle of the hammer, and with it the membrana tympani, inward, thereby putting the latter on the stretch. This action can be readily seen on a specimen where the canal of the muscle and the cavity of the tympanum have been opened from above. By pulling upon the tendinous fibres of the muscle within the canal, the membrana tympani becomes tense. As the point of insertion of the muscle lies but a trifle lower down than the axis-band of the hammer, the band itself will also at the same time be stretched inward, and especially the posterior portion of it, the ligamentum mallei posticum, which lies very nearly in the same line with the line of traction of the tensor tympani. The position of the hammer is thus made quite firm, even though the tendon be but moderately tense. We must remember here that a slight traction, when made at right angles to the length upon an inextensible cord which is already tense, can very materially increase its tension; and also that during life a muscle in a state of rest must be considered as a very yielding, though always slightly tense elastic band, whose tension can be very considerably increased by active contraction. Aside from the fact that the tensor tympani, on account of its penniform construction, is, mechanically speaking, equivalent to a muscle of much greater diameter and shorter length of fibre, we can consider its simple elastic traction, without the occurrence of any active contraction whatever, as a pretty important power.

In this way it is clear that the hammer, so long as it retains its natural attachments—although these be but yielding bands—and even after the division of the stapedo-incudal joint, is capable of only a limited motion in the direction of rotation about the above-mentioned axis; and that any attempts to move it in a different direction meet with very strong resistance. Anteriorly its axis is held securely by the ligamentum

anterius, and the processus Folianus, which is embedded in its meshes; posteriorly by the posterior fibres of the ligamentum externum: the two together we have called the axis-band of the hammer. This is always pretty tense, even after the tendon of the tensor tympani has been divided; but if the latter draws upon the axis-band at a right angle, its tension then becomes very great.

The hammer thus fastened, possesses, besides the tendon of the tensor tympani, the following bands, capable of restraining any rotation of the handle outward: 1. the middle and anterior fibres of the ligamentum externum, 2. the ligamentum superius, 3. the upper fibres of the ligamentum anterius. The membrana tympani itself acts as a band of restraint against too strong rotation of the handle of the hammer inward.

As far as the slight yielding capacity of the axis-band and of the upper (relatively lower) fibres of the ligamentum anterius will allow, the head of the hammer can incline forward and backward, or rotate about a vertical axis. Nevertheless, when the hammer is connected with the anvil these motions are thereby still further limited. Still, we shall see that the motion of the hammer and anvil together requires a certain amount of yielding capacity on the part of the axis-band.

§ 4.

Attachments of the Anvil.

The body of the anvil is united to the hammer through the medium of a joint. Its long process extends downward, and has at the end (which is bent somewhat inward) a small articular surface for the stirrup. The short process extends backward, and its extremity, on whose lower aspect there is a small incomplete articular surface, lies in an appropriate hollow cut out of the bony wall of the tympanum, at a point where the cavity of the latter merges into that of the cells of the mastoid process. The capsule of this joint, on it supper surface at least, is composed of strong tendinous fibres which extend inward, backward, and outward from the short process (see Fig. 5, *bi*). In the same figure, *i* represents the body of the anvil, and * the capsular ligament of the malleo-incudal joint.

The shape of the last-named articular surface is usually described as resembling a saddle; it must be remarked, however, that, unlike the saddle, not only the convex sides come together to form a ridge that is almost sharp, but also the concave; and the union of the two forms a continuous and almost flat surface on either side of the ridge. In order to gain a clear idea of the mechanism of this joint, it is better, I believe, to make use of a different comparison than that of the surface of a saddle. It is, in fact, like the joint used in certain watch-keys, where the handle cannot be turned in one direction without carrying the steel shell with it, while in the opposite direction it meets with only slight resistance. As in the watch-key, so here the joint between hammer and anvil admits of a slight rotation about an axis drawn transversely through the head of the hammer toward the end of the short process of the anvil; a pair of cogs oppose the rotation of the manubrium inward, but it can be driven outward without carrying the anvil with it.

If such a joint had to be constructed of metal, we should make use of screw-surfaces. A hollow cylinder, cut as A is represented in Fig. 6, and upon which the piece B (marked in dotted lines) fits, would represent the normal shape of such a joint. It is clear that A and B, revolving in the direction of their respective arrows, must necessarily strike against each other with their cogs *a* and *b*; hence, in this direction their rotation is limited. In the opposite direction, however, their rotation is free, and is accompanied by a slowly increasing separation of the two cylinders. The mechanic, in making such a joint, usually employs a hollow cylinder, because in the neighborhood of the axis, were the cylinder solid, the screw surface would incline upward at a pretty steep angle, like the inner border of a winding staircase, and hence would be difficult to execute. The articular ends of the bones, which are covered with a layer of elastic yielding cartilage, filling out all the irregularities of

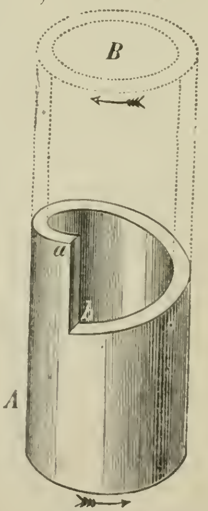

Fig 6.

surface, show, as a general rule, only a modified form of the above geometrical outlines, one where the margins are rounded off, etc. The periphery of the malleo-incudal joint is not a regularly formed screw outline. If we imagine it rolled out upon a plane from the cylindrical circumference of the joint, it would have more or less the form represented in Fig. 7, where the ends a_0 a_1 are naturally continuous. Near the axis of the joint the surface assumes, not exactly the shape of a screw as in a winding staircase, but rather more that of a cone. If we suppose straight lines to be drawn from a point in the axis of the cylinder to all points of the line of circumference a_0 a_1, we shall obtain an approximate idea of the shape of the joint in question. In the case of the hammer, we would have to place the apex of the cone

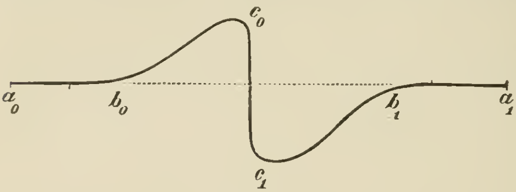

Fig. 7.

thus formed somewhat lower down than the straight part of the line b_0 a_0 a_1 b_1, so that this portion of the surface of the cone would be concave on the hammer, while on the anvil the corresponding part is convex. Such a joint may therefore be said to consist of four nearly plane surfaces, which come together at its centre, and along its border show the following margins: 1. c_0 c_1, 2. c_0 b_0, 3. c_1 b_1, 4. b_0 a_0 a_1 b_1, while the upward-turned surface of the joint, like a saddle, shows two salient borders c_0 and b_1, and two reëntering b_0 and c_1.

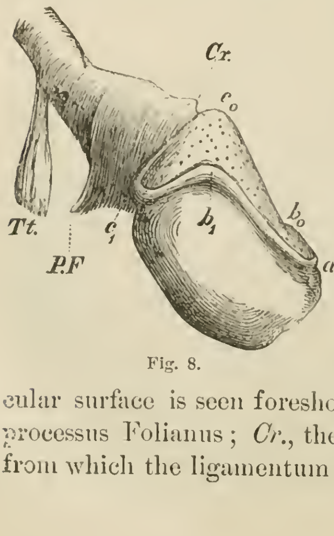

Fig. 8.

In Fig. 8, the hammer is represented as it appears from above and inside. The letters ab_0 c_0 b_1 c_1 have the same meaning as in Fig. 7. The flat portion of the articular surface is seen foreshortened. *P.F.* is the stump of the processus Folianus; *Cr.*, the commencement of the bony crest from which the ligamentum mallei posticum arises; *T.t.* is the

tendon of the tensor tympani. As can be seen, the part a a_1 lies on the upper side of the malleo-incudal joint, while the line of junction of the two cogs c_0 c_1 lies lower down between the handle of the hammer and the long process of the anvil; the cog c_0 of the hammer lies on that side of the joint which is turned toward the membrana tympani; the cog of the anvil, on the median side. The handle of the hammer cannot therefore rotate inward without carrying the anvil with it; the limit of rotation outward is governed by the flexibility of the ligaments and cartilaginous covering of the articular surfaces.

In the articular surfaces of both ossicles the point a_0 a_1 lies at a greater distance from the axis than the border of the cog c_0 c_1; in other words, that portion of the surfaces which is nearly plane is greater than the surfaces of the cogs, and consequently the entire surface of the joint is elliptical in shape, with the longer axis running vertically. It should be stated, furthermore, that the apex of the cone-shaped articular surface (we have assumed the cone to be the type of this joint surface) does not come to a point, but is rounded off, as in a saddle.

A conical surface, such as may be made by aid of Fig. 7, cannot be made to revolve upon its exact counterpart without loss of contact at some point; for when the two screw lines b_0 c_0 and b_1 c_1 glide upon each other at the periphery of the joint, the central portion of the joint and the more level portion will separate each from the corresponding portion, leaving the two bones in contact only at the above-mentioned screw lines. Since the working space in such a motion is very small, therefore we may readily perceive that in a fresh specimen a compressible cartilage may quite fill it.

This peculiar mechanism of the joint can be readily seen in dry specimens of the ossicles; one match may be fastened with sealing-wax to the head of the hammer, just above and in the same direction with the processus Folianus, and another to the anvil, at the end of its processus brevis, and in the same direction with it; by using these matches as handles, and bringing the two articular surfaces together, the hammer may be rotated about its match as an axis, while the anvil is simply pressed lightly against it. If the hammer is rotated in the

direction from the head toward the short process and handle, it will grasp the anvil very firmly, and compel it to follow. When the hammer is rotated in the opposite direction, the two articular surfaces at once separate from each other and the anvil remains stationary.

The two articular surfaces are kept in contact at their peripheries by a capsular band which is inserted into grooves in the bone at various points surrounding the joint. This capsular band is not very strong; it tears when the ossicles are exposed to comparatively slight strains. Of this band the strongest fibres are those which proceed from the cog of the hammer; at this point, too, a few fibres of the ligamentum externum of the hammer pass over to the anvil. The length of an excursion of the malleo-incudal joint, measured at the lower end of the long process of the anvil, amounts to about half a millimetre; and this point being nearly 6 millimetres distant from the axis of the joint, the rotation of the two ossicles upon each other will hardly reach 5°.

So long as the hammer and anvil retain their natural connections with each other, and with the petrous bone, (the anvil, however, being separated from the stirrup,) they may rotate conjointly in such a manner that the handle of the hammer, the long process of the anvil, and the membrana tympani will all at the same time move either inward or outward. The hammer by itself would rotate about its axis-band as axis, but owing to its connection with the anvil, this mode of rotation is somewhat modified. It will be noticed, in the first place, that the short process of the anvil (Fig. 5, *bi*) is attached to the petrous bone at a point considerably to the inside of the prolonged axis band of the hammer. In a simple rotation about a fixed axis, only those points are at rest which are in the very axis of rotation. The distance, furthermore, of any individual points in the rotating body, that are situated outside of the axis of rotation, from an external fixed point, cannot remain unchanged during rotation; in the present case the fixed point is where the short process of the anvil is attached to the petrous bone. The only exception to this, however, is in the case of infinitely small rotations, where those points of the rotating body, that lie in a plane carried

through the axis of rotation and the external fixed point, remain at the same distance from that fixed point. This is not the case with the head of the hammer, which is situated above the axis of rotation; hence, when the handle of the hammer is rotated inward, the head will be lifted away from the point where the short process of the anvil is bound down. Now, inasmuch as the anvil is, so to speak, suspended by pretty short and rather unyielding bands between the head of the hammer and the above-mentioned point of attachment, and inasmuch as it maintains an immovable position, the rotation of the handle of the hammer inward must be accompanied by a slight inclination of the head backward toward the anvil, simultaneously with a motion of the handle forward. That such an inclination of the head backward actually does take place, may be inferred from the tension that is visible during rotation in the capsular ligament at the upper side of the malleo-incudal joint, in the uppermost fibres of the ligamentum mallei anterius, and in the bands which give firmness to the tympano-incudal joint. By pressing the membrana tympani inward with the head of a pin, we can see, with an ordinary magnifying-glass, that these two capsular ligaments become tense the moment the pressure is exerted. Furthermore, if we press with a needle upon the short process of the anvil, while the membrana tympani is thus being pushed inward, we shall feel and see that this process does not lie in contact with the bottom of the shallow groove in which it fits, but is lifted slightly above it, and that, when we bring this process into contact with the groove by pressure from above, the upper, satin-like accessory ligaments of the joint become relaxed, and are thrown into folds. On the other hand, the tip of this short process lies in contact with the wall of the tympanum, which rises up on its outer side; its connections, moreover, with this outer wall are such that it can glide a little upon the latter in a vertical direction. At the same time, the anvil is held suspended free in the air by the hammer, so that in its normal position it comes in contact with bone only at the outer side of the end of the short process. If, however, the handle of the hammer and the membrana tympani are forced outward, the tip of the short process of the anvil will glide

down into its appropriate groove in the bone and will press upon it with its under surface.

With a joint of this nature, a slight displacement must necessarily take place between the hammer and anvil whenever the end of the handle of the hammer is driven inward. If we imagine for a moment the anvil and hammer immovably joined together, and the latter made to revolve about its axis-band, the end of the short process of the anvil, lying out of the line of this axis, will necessarily be lifted from its bed whenever the membrane of the drum is driven inward. To lower the end of the short process again and bring it back to its place, the anvil must rotate slightly on the hammer. A limited degree of such motion is possible from the saddle-shape of the malleo-incudal joint. At the same time the long process of the anvil approaches slightly the handle of the hammer. This latter motion can be observed on specimens where the stapedo-incudal joint has been severed, while all the other connections remain undisturbed. It is just in this position of the two ossicles, moreover, that the lower parts of the joint surfaces, which are here armed with teeth, press most upon one another (see Fig. 8). This can be ascertained to be a fact by placing the two dry ossicles together in the manner described above and noticing in which position they fit the closest.

The nature of this joint further requires a slight displacement on the part of the hammer. If its head must incline toward the anvil, it cannot do so without drawing the axis-band away from a straight line. The anterior side of the neck with the processus Folianus and ligamentum anterius would have to be lifted up, while the posterior side of the neck with the posterior fibres of the ligamentum externum were being drawn down. The first of these motions, however, could hardly take place on account of the spina tympanica posterior, which lies immediately above the processus Folianus, and against which the latter would at once strike. All the more marked, then, would have to be the sinking of the posterior side of the neck, and, with it, of the entire hammer, thus causing a greater tension of the fibres of the lig. mallei post. which run from the hammer in a backward and somewhat upward direction.

These views are in harmony with a short notice recently published by Politzer.* He attached fine glass rods as levers to the ossicles in order that he might determine more accurately their individual axes of rotation. He put the membrana tympani in motion by compressing the air in the external auditory canal. In this way he ascertained that the axis of rotation of the hammer runs through the end of the processus Folianus, and that of the anvil through the extremity of its short process, but that both of these axes are movable.

It appears to me that it is in a great measure owing to the slight changes in the axis of the hammer, caused by the peculiar nature of the anvil's attachments, that the navel of the membrana tympani always moves in a normal direction relatively to the plane of insertion of this membrane. For inasmuch as the axis-band of the hammer is situated obliquely to the plane of insertion of the membrana tympani, every motion of the manubrium mallei inward would also at the same time produce a slight displacement of the navel of the membrane backward. But, through the medium of the anvil, the head of the hammer is at the same time drawn backward, thereby causing a motion of the handle forward in the opposite direction.

Again, the navel of the membrana tympani lies at a greater distance from the plane of insertion of this membrane than the axis of rotation of the hammer, (excepting, possibly, its extreme anterior end at the spina tympanica,) so that every motion inward of the manubrium would also carry the navel of the membrana tympani a little upward (that is, in the direction of the head of the hammer). This upward motion is counteracted, as we have just described above, by the circumstance that the hammer as a whole is drawn somewhat downward by the anvil (in a rotation inward of the manubrium).

In this manner, therefore, both these deviations are corrected in the motion of the navel of the membrana tympani; the only kind of motion that remains to it, then, is that which takes place in a direction at right angles to the plane of insertion of the membrana tympani.

At the same time it will be apparent that in these displace-

* Wochenblatt der K. K. Gesellschaft der Aerzte. Wien, 1868, Januar 8.

ments of the hammer its short process must glide a little upon the membrana tympani. Such a gliding motion is rendered possible by the peculiar manner (described by J. Gruber) in which these two parts are attached the one to the other.

I would like, furthermore, to call attention to the fact that, by the contraction of the tensor tympani muscle, all the bands which give firmness to the position of the ossicles are rendered tense. This muscle, in the first place, draws the handle of the hammer inward, and with it the membrana tympani. At the same time it pulls upon the axis-band of the hammer, drawing it inward and putting it upon the stretch. Another effect, as we have shown, is to draw the head of the hammer away from the tympano-incudal joint, to tighten all the ligaments of the anvil, those toward the hammer as well as those at the end of its short process, and to lift the latter up from its bony bed. In this way the anvil is brought into the position where the cogs of the malleo-incudal joint fit into one another the tightest. Finally, the long process of the anvil is compelled to perform a rotation inward in company with the handle of the hammer; in so doing, as we shall see further on, it presses upon the stirrup and drives it into the oval window against the fluid of the labyrinth.

In this respect the construction of the ear is very remarkable. By the contraction of the single mass of elastic fibres constituting the tensor tympani (whose tension, besides, is variable and may be adapted to the wants of the ear) all the inelastic tendinous ligaments of the ossicles are simultaneously put upon the stretch.

The only ligament that is thereby relaxed is the ligamentum mallei superius, whose influence as a ligament is essentially in the same direction as that of the tendon of the tensor tympani.

Hence, if we examine a freshly prepared specimen of the ear in which the rigor mortis is still manifest in the tensor tympani muscle, we shall find every thing in the tympanum stiff and unyielding; whereas later, if we attempt to separate the different parts, we shall find that almost all the bands and ligaments of the ossicles are loose and relaxed; in fact, without a careful in-

vestigation of the relations of these parts, we should be at a loss how to harmonize the two conditions.*

§ 5.

The Movements of the Stirrup.

The joint of the anvil and stirrup resembles the flat segment of a sphere which is convex toward the stirrup. The capsule is soft, and interwoven more with elastic fibres than the two other articulations; on its inferior side there are compact fibres which, when the anvil is drawn upward, are put on the stretch and carry the anvil with them, but when the reverse movement takes place, are folded together so that the anvil does not follow so closely as before.

The base of the stirrup is surrounded by a lip of elastic fibro-cartilage resembling the lips of cotyles and of larger articulations: it has a breadth of 0.7 mm. The union between the base of the stirrup and the wall of the labyrinth appears to be formed by means of the periosteum of the vestibule, which periosteum is extended over the base of the stirrup. The fibrous lip of the stirrup is not attached to the sides of the fenestra ovalis.

The mucous membrane of the cavity of the tympanum extends also over the outer side of the joint. The attachments of the base of the stirrup along its straight edge are more tense on the inferior than on the superior surface, and most compact at the posterior end. Now, if you apply a pin to that side of the

* In regard to the question when the tensor tympani contracts, I would confirm in this place the recently published observation of Politzer, that it occurs during the act of yawning. Before hearing of his experiments, I had already noticed that whenever I attempted to restrain the motion of the jaws during the act of yawning, I would first hear the well-known snapping noise, indicative of the opening of the tube; then, at the acme of the yawn, I would notice, in addition to the sense of tension in the ear, a loud muscular noise, greater even in intensity than that produced by the most powerful contractions of the masticator muscles during closure of the meati, and certainly greater than when the meati are open. All sounds from without appeared at the same time to be muffled. From these observations I concluded that contraction must have been excited in a muscle whose oscillations are communicated to the organ of hearing with far greater distinctness than those of any other muscle: I refer to the tensor tympani.

base of the stirrup which is toward the vestibule and press it outward, it will, though separated from the anvil, make at the same time a lever-like movement by which its head is pushed downward and backward. If you employ a fine sewing-needle as a *lever of sensation* and press it into the base of the stirrup, the lever-like movement can be still better observed. In other respects the mobility of the base of the stirrup is very slight. I have calculated it partly by direct observation and partly from the movement of the water of the labyrinth. For the purpose of direct calculation I employed a preparation in which the cavity of the tympanum and the vestibule had been opened from above.

The preparation was held firmly in a vice in such a position that the base of the stirrup looked downward. The point of a fine sewing-needle was then inserted into the membrana obturatoria, near the anterior limb of the stirrup; the needle had for its second fulcrum the sharply cut edge of what remained of the osseous wall between the cavity of the tympanum and the labyrinth. This point, 3.8 mm. removed from the point of the needle which was fixed in the ligamentum obturatorium, served as the centre of motion for the lever-like movements. The free portion of the needle, which was horizontal, formed the second longer arm of the lever, (length, 23 mm.). The point of this longer arm moved backward and forward 0.20 mm., when the stirrup was pressed inward and outward by means of a needle applied to its base; and .15 mm., when the pressure was made by alternately condensing and rarefying the air in the external meatus, whereby the movement of the membrana tympani was transmitted through the other bones of the ear to the stirrup.

Now, since the movements of the stirrup seemed magnified, at the free end of the needle, $\frac{23}{3.8}$, therefore the displacements of the stirrup itself amount in these cases only to 0.033 and 0.025 mm. After frequent repetition of the experiment, by which the ligaments were thoroughly stretched, the amplitude of displacement increased to 0.056 mm. In another preparation only the superior semi-circular canal of the labyrinth was opened, according to Politzer's plan, from the upper side of the tem-

poral bone. Into the opening thus made was inserted a slender glass tube, whose transverse section was found, by calibration with quicksilver, to be 0.228 of a square millimetre. The vestibule and a portion of the tube were filled with water.*

The movements of the bones of the ear produced by forcing air into the external meatus caused the fluid in the tube to rise 0.9 mm. Now, since the diameters of the fenestra ovalis were found equal to 1.2, and 3 mm., therefore the surface of the fenestra ovalis is nearly 12.4 times as large as the transverse section of the glass tube. The mean amplitude of the excursion of the base of the stirrup must then be $\frac{1}{12.4}$ of that of the fluid in the tube, which is 0.0726 mm. According to the highest calculation, the excursions of the stirrup amount to $\frac{1}{18}$ and $\frac{1}{14}$ mm.

The relation of the stirrup to the anvil is such that if the handle of the hammer be drawn inward, the long process of the anvil presses firmly against the knob of the stirrup; the same takes place if the capsular ligament between both be cut through.

If the manubrium be moved outward as far as the ligaments of the hammer will allow, then, in case the capsular ligament be cut, the long process of the anvil will recede from $\frac{1}{4}$ to $\frac{1}{2}$ mm. from the stirrup. With this position of the hammer, if the handle of the anvil be pressed back against the stirrup, it will remain in this position without springing back; at the same time the cogs of the joint of the hammer and anvil become separated entirely, and there is no force present sufficiently powerful to draw the anvil back. In the normal condition of the articulation of the anvil and stirrup, the point of the handle of the anvil remains always attached to the stirrup; but it follows, from the already-mentioned facts, that the anvil exercises no strain upon the stirrup when the handle of the hammer is driven

* In order to render it air-tight, I first dried the bone as much as possible with blotting-paper; then I applied a red-hot iron wire to the margin of the opening in the semi-circular canal, and upon this spot I placed immediately a drop of hot cement made of wax and resin; in this the glass tube was fastened. Finally, the preparation was set in a bowl of water of sufficient depth to cover the free end of the glass tube, and then the whole was placed under the air-pump. On exhausting the air in the chamber, the air in the vestibule escaped through the tube, and then water took its place.

outward, since the handle of the anvil, even when the articulation is severed, can remain in contact with the stirrup without being drawn outward with the handle of the hammer.

This arrangement has this important result, namely, that by means of an increase of pressure in the cavity of the drum or diminution of pressure in the meatus, the membrana tympani and the hammer can be perceptibly driven outward without the stirrup incurring the danger of being torn from the fenestra ovalis. The membrana tympani serves as a very powerful restraint to the reverse movement of the hammer.

Since the point of the long process of the anvil, seen from the axis-band, is inclined still further backward than the point of the handle of the hammer, therefore the former rises when pressed inward more than the latter, and the elevation is not entirely compensated for by the slight depression of the hammer already mentioned ; in short, the driving of the membrana tympani inward causes the point of the head of the hammer to be driven outward and at the same time slightly elevated.

This agrees with the corresponding movement of the tirrup, whose knob-like head rises slightly when the stirrup is pressed inward in consequence of its unequal attachment to the upper and lower border of the fenestra ovalis. This lever-like movement has already been observed and described by Henke,* Lucae,† and Politzer.‡ In reply to the first, I will only remark that the lever-like movement of the stirrup is by no means its only one ; that "perhaps" one border of the plate of the stirrup is not moved inward while the other moves outward. Looking at the base of the stirrup from the vestibule, we can much more easily recognize the fact that both borders are driven outward and inward simultaneously ; the superior, however, more than the inferior.

The apparent discrepancies between the observations of Lucae and Politzer, in respect to the effect which an increased pressure of air in the cavity of the tympanum has upon the

* Der Mechanismus der Gehörknöchelchen, in der Zeitschrift für rationelle Medicin. 1868.

† Archiv für Ohrenheilkunde. Vol. iv. pp. 36, 37.

‡ Wochenblatt der K. K. Gesellschaft der Aerzte. Wien, 1868.

stirrup and the water of the labyrinth, are to be explained by supposing that Lucae observed the lever-like movement of the stirrup, and Politzer the oscillation which takes place in the water of the labyrinth when the stirrup is pressed inward. Neither is it always necessarily in the same degree; at least not in this case, because the pressure of the air through the fenestra rotunda can also cause increased pressure in the labyrinth.

§. 6.

The Concerted Action of the Bones of the Ear.

If we suppose the hammer and anvil so united that their cogs press against one another and both move like one compact body, exerting a pressure upon the point of the handle of the hammer, which is continued inward and transmitted from the anvil upon the stirrup, then the system of the two ossicles can be considered as a one-armed lever whose fulcrum lies where the point of the short process of the anvil presses outward against the wall of the cavity of the tympanum. The tip of the handle of the hammer represents the point of pressure, and the point of the handle of the anvil the other point which resists this pressure. These three points lie in fact very nearly in a straight line, so that the point of the anvil and stirrup recedes only very slightly inward from the straight line of union between the tip of the handle of the hammer and the outer side of the articulation of the anvil with the tympanum. This can easily be seen in preparations where the natural union of the bones is still preserved. Fig. 9. represents both bones in the position where their cogs are attached to one another; *a a* is the straight line which passes through the three points mentioned above;

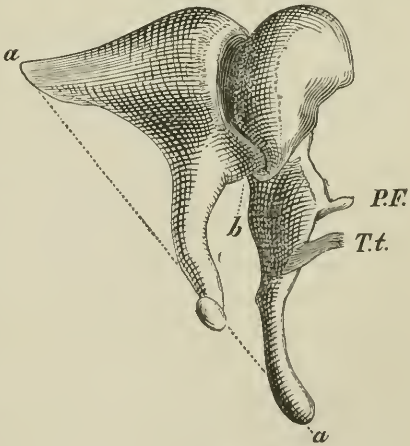

Fig. 9.

P. F., the stump of the processus Folianus; *T.t.* the tendon of the tensor tympani; and at *b* we have the cog of the anvil. In this preparation I found the entire length of the lever to be $9\frac{1}{2}$ mm., the shorter arm between the two points of the anvil $6\frac{1}{3}$ mm., so that the latter is $\frac{2}{3}$ of the length of the longer arm. Hence, when the hammer and anvil lie close together, the excursion of the tip of the handle of the anvil will amount only to $\frac{2}{3}$ of that of the handle of the hammer, but the amount of the pressure which the former exerts upon the stirrup will be $1\frac{1}{2}$ times as great as the force which is exerted against the handle of the hammer.

Since the three points of the lever lie in a straight line, therefore the pressure is quite independent of the position of the remaining parts of the ossicles—presupposing only that the latter maintain a position in which the articular surfaces press firmly against each other.

This result will be obtained in the following manner: While the membrana tympani is being driven inward, the hammer will rotate about an axis which is inclined obliquely (30°) toward the plane of insertion of the membrana tympani, and its head will recede from the tympano-incudal joint, thus putting the malleo-incudal capsular ligament on the stretch. Now, since every attempt to rotate the ossicles in such a manner that the cogs will press against each other, produces instantly a considerable separation of the articular surfaces, therefore the already tense fibres of the capsular ligament serve to resist any force capable of producing this result. On the other hand, if the membrana tympani be driven outward, the capsular ligament becomes relaxed and yields to such an extent as to allow of a slight separation of the articular surfaces, like that which happens when the cogs are separated. The remaining displacements, which the malleo-incudal joint admits of, do not cause the above-mentioned three points of the lever to depart from the straight line. One of the axes of rotation of this saddle-shaped joint passes through the tip of the handle of the hammer; the other is perpendicular to the plane which passes through the three points and the articulation, and consequently separates the long process of the anvil from the handle of the hammer.

In the above-mentioned experiments, where, as a general thing, the stirrup has been set in motion by exerting pressure upon the hammer or upon the membrana tympani, we have seen that the amplitude of the excursion becomes slightly diminished, that is, to about $\frac{2}{3}$ of its magnitude. In order to determine the firm ness of the mechanism I tried the reverse experiment and endeavored to measure the extent ot the excursion of the hammer by pressing the base of the stirrup outward and thus setting the hammer in motion. In this case, of course, the only movements of the hammer to be taken into consideration are those which cause it and the anvil to remain in close contiguity at the point of contact. (The preparation used in this experiment has been already described as having a tube inserted into the vestibule.) Having cemented a glass thread 59 mm. long to the head of the hammer, I endeavored to find how much motion could be produced in the hammer by alternately forcing fluid through the glass tube and then withdrawing it from the same. The excursion at the tip of the glass thread amounted only to about $\frac{1}{2}$ mm. If 4 mm. be the distance from the axis of rotation to the point where the glass thread is fastened to the head of the hammer, then the length of the lever is 63 mm. and the above-mentioned excursion of $\frac{1}{2}$ mm. corresponds to a rotation of about half a degree. For the tip of the handle of the hammer, whose distance from the axis-band amounts to $4\frac{1}{2}$ mm., this gives, on the other hand, an excursion of only $\frac{1}{28}$ mm., an amount which is about the same as the mean value of the excursion of the base of the stirrup. Theoretically we should expect a somewhat greater value for the excursion of the handle of the hammer. Taking into consideration the diminished tension of animal tissue after death, and the want of elasticity, namely, of the tensor tympani, we cannot well expect in the united action of the bones of the ear the same precision which we find in the living subject. In this way the transmission of the light movements of the anvil to the hammer may be impaired. *

* I will remark in this connection that the transmission of the movements of the membrana tympani to the water of the vestibule was also perceptibly impaired at the time when I made the above described experiment. I obtained

These different attempts at measurement coincide thus far in showing that the displacements of the stirrup and hammer, as long as the two remain firmly connected, are limited to amplitudes each of which is smaller than a tenth part of a millimetre.

On the other hand, if we put the hammer in motion by forcing air into the external meatus and then withdrawing it, the glass thread attached to the ossicle indicates much greater excursions; its point moves backward and forward 5 mm., while before (as already mentioned) it experienced, in a direction from the stirrup outward, only a displacement of ½ mm.

The excursion which the hammer can make without the anvil is nearly nine times as great as that which the two together can accomplish. This kind of movement is not transmitted to the water of the labyrinth, excepting of course the slight changes in pressure which the changed tension of the articular ligaments, or the rubbing of the articular surfaces of the malleo-incudal joint upon one another, are perhaps sufficient to produce in the water of the labyrinth when the cogs of the articulation are no longer in contact. If you force air into the cavity of the tympanum of your own ear, you will hear feeble tones issuing from the middle and upper portions of the scala, almost if not quite as distinctly as usual; on the other hand, it is very apparent that we hear the same tones, when they are given forcibly, much more distinctly when the pressure in the cavity of the tympanum is uniform than when it is increased. This, I think, shows that the articular surfaces of the hammer and anvil can adhere together and be firmly united by means of motion on one another, similar to that which takes place in anatomical preparations when the joint of the anvil and stirrup has been cut through and the rarefaction of the air in the meatus auditorius has caused the hammer to be drawn outward. The anvil then is also drawn outward; but if we turn it by means of a needle, so that its long process again touches the stirrup, it will, as mentioned above, still remain fixed in this

only 0.4 mm. elevation in the manometer, whereas, on the day before, when I had filled the vestibule with water under the air-pump, the elevation amounted to 0.9 mm. It is to be hoped that some anatomist, who has an abundance of suitable preparations at his command, will repeat these experiments. Of course the specimens should be as fresh as possible.

position. Friction will also cause the anvil to adhere firmly to the hammer in the position already given, in opposition to the tension of the ligaments or any other mild forces, and that too when the vibrations of sounds are feeble. More powerful forces or concussions will of necessity cause the two bones to slide upon one another, and strong vibrations of sound in such a position of the bones will be very perceptible.

I have used in these experiments a watch and a tuning-fork; striking the latter lightly, I held it so far from the ear that the beats, which the rotation of the fork upon its long axis produced, were still perceptible. We hear them just as distinctly when the membrana tympani is distended, provided they belong to the upper octaves of the scale, and very nearly as distinctly in the middle octaves. The deeper tones are, of course, considerably weaker. On the other hand, a tuning-fork of a higher pitch, when struck forcibly and held before the ear while the membrana tympani was distended, showed a very perceptible crescendo, just as we restored the equilibrium of the air by the motion of swallowing.

I wish to call attention, in this connection, to another phenomenon whose explanation, I think, can be deduced from the mechanism already described. If we take a tuning-fork which consists of a single piece of steel, and which therefore has nothing about it which can give a rattling sound, and, after having struck it forcibly, hold it near the ear so that the sound can be heard very distinctly, the character of the tone becomes sharp, and we hear distinctly jarring sounds similar to what is heard in musical instruments when something is loose, or from a tuning-fork when pressed rather lightly upon a sounding-board. These jarring sounds result from the slight shocks which a vibrating body makes upon a body at rest or vibrating in a different manner. These blows are repeated regularly and produce sound; but inasmuch as they correspond to an interrupted periodical movement, the sound possesses very many overtones and is harsh in character. Such tones occur, as is well known, in the ear itself as the result of very loud sounds. We can hear also from a *B* tuning-fork of 116 vibrations a jarring sound so distinctly that it resembles a buzzing in the ear. This jarring

4

tone is very distinct and strong when the pressure of the air in the cavity of the tympanum is equal to or less than that of the atmosphere, and when the cogs of the hammer and anvil are closely united; but it disappears when the air is driven into the cavity of the tympanum and the cogs are consequently separated. I think, therefore, that we are justified in concluding that this jarring tone is caused by the cogs.

When the excursions of the membrana tympani are very great, and during the outward phase of the vibration, the anvil is not driven outward with any considerable force, and cannot therefore follow perfectly the excursions of the hammer; the result of which is that they are separated, and that during the next vibration inward the anvil receives a blow from the returning hammer. This mechanism is also well adapted to the production of combination tones,* and the peculiar sensation of buzzing in the ear resulting from the combination tones of two strong soprano voices, when thirds are sung, can, I think, be referred to this jarring which takes place between the hammer and anvil.

This phenomenon is also of great importance in its relation to the sensation which harmony produces in the ear, since strong tones which take place outside of the ear, and without overtones, must of necessity develop harmonious overtones in the ear. In this way sounds with harmonious overtones, which correspond to a regular periodical movement of the air, acquire a natural preference over those with unharmonious overtones, especially as the whole doctrine of conferences becomes, through this circumstance, independent of the overtones connected with external sound. The jarring tones can be much deeper than the exciting tone, if the vibrating body falls back only after the expiration of the vibrations, and hence receives another blow.

To this class, I believe, belong also certain deep, harsh sounds which we hear when the shrill high notes of the upper octave ($a_4 - g_4$) are sounded very distinctly. It is probable that an unusually strong vibration is produced at the same time in the surface of the membrana tympani, judging from a certain buzzing, tickling sensation which is felt in the deep parts of the

* Vide Lehre von den Tonempfindungen, pages 233-236.

ear. The apparatus described further on (Fig. 11) is particularly adapted to the production of such tones.

I will mention here that I have made an enlarged model of the apparatus of the cavity of the drum, in order to prove the completeness and correctness of the explanation just given. The bones of the ear are made of wood, the membrana tympani of glove-leather, cut in such a way that a seam shall run along the handle of the hammer where the leather is attached to that ossicle. By this means we can give to it its conical form. An opening of suitable form, cut in a board, and having beveled edges to which the edges of the artificial membrana tympani are fastened, represents the inner end of the external meatus. On the outside of the board a tin ring is fastened, which surrounds the already mentioned opening. To this, finally, a tin cover having a gutta-percha edge is fitted, like the covers of hermetically sealed fruit-jars. Now, if we place this cover so that a portion of the gutta-percha rim remains between it and the tin ring, condensation of the air may be produced on the outer side of the artificial membrana tympani, which will act upon the bones of the ear.

On the inside, near the upper and anterior edge of the opening, a thin piece of wood with a projecting point is fastened, which latter represents the spina tympanica major. A hemp string, which is attached to the latter, penetrates the hammer and passes around it, then passes through the board at the posterior superior edge of the opening. This string, which is intended to represent the axis-band, can be rendered tense by an ordinary screw-eye. The tendons of the ligamentum externum and the ligamentum anterius mallei, which pass from the spina upward, can be represented by other strings, which of course must be accurately applied and provided with screw-eyes to vary their tension. Finally, the tendon of the tensor tympani can be represented by a silk thread which passes through an iron ring made fast to a small wooden pillar, and then is connected with a tense gutta-percha band. I first spread warm sealing-wax upon the articular surface of the hammer, and endeavored as far as possible to give to the former, before it had grown cold, the corresponding form; then I laid soft, hot sealing-wax upon the

articular surface of the anvil, and, after having covered the articular surface of the hammer with tin-foil, I pressed the two together. The tin-foil then adheres to the anvil. Now, before the sealing-wax had become quite cold, I made a twisting movement with the anvil, similar to that which occurs between these bones in the ear, in order to render these surfaces capable of sliding upon one another. After the articular surface of the anvil had become cold, it served as a form upon which to mould the surface of the hammer (which must be heated and covered with tin-foil) and render it capable of gliding over the articular surface of the anvil. This experiment was repeated alternately with one and the other articular surface until the two moved sufficiently easily upon one another. It was, of course, necessary not to make any sliding movements strong enough to disturb the cogs. In this way I succeeded finally in obtaining a good articulation. The capsular ligament was constructed of loops of thin elastic India-rubber cord, which were fastened to the anvil and could be drawn over and attached to the hammer by means of small hooks made of pins, thus holding the two bones together by a very slight elastic pressure.

The articular ligament of the short process of the anvil was represented by a loop of silk threads which passed through a hole in the anvil. This band can be loose, but it is of importance that the point of support of this part of the anvil upon the outer wall of the cavity of the drum should be represented in the model.

Simple contact is sufficient to represent the union between the long process of the anvil and the stirrup, or a loop of silk threads can be employed. The former is quite sufficient for giving direction to the blows above described.

The fenestra ovalis was cut in a thin piece of board, which was held parallel to the larger board by means of small wooden pillars. This board consisted of two plates screwed together, between which was a thin layer of gutta-percha, representing the membrane of the fenestra ovalis. The foot-plate of the artificial stirrup was likewise double and had an interposed layer of India-rubber, and the whole was fastened together by screws.

Such a model is very useful, partly in demonstrations and partly to show clearly what part the individual ligaments and also the articulations play in connection with the attachment of the bones of the ear; for all these different parts can be separated, and each ligament can be made tighter or looser. Moreover, this model transmits with great facility to the stirrup the small blows which are directed on the outside, immediately upon the manubrium or upon the already mentioned air-tight cover; this can be felt when the finger is placed over the base of the stirrup and over the plate in which it rests, and is also recognizable in the bounding up of light bodies which have been laid upon it.

The diameters of the artificial membrana tympani are 80 and 120 mm. The remaining parts are constructed according to this measurement. All the statements contained in the foregoing description, in regard to the mobility and mode of attachment of the parts, I have tested and found confirmed in this model.

§ 7.

Mechanism of the Membrana Tympani.

The membrana tympani is to be considered as a tense membrane, which, however, differs essentially from those which have been hitherto studied in acoustics, in the fact that it is curved. Its tension is modified by the handle of the hammer which draws it inward, and which is itself retained in this position by means of ligaments of attachment, and by the elasticity of the tensor tympani. If the radial fibres of the membrana tympani were not united by transverse ones, they would be stretched in a straight line. In point of fact, however, they maintain a curved shape with the convexity looking toward the meatus; hence we conclude that the radial fibres are drawn toward one another by circular fibres, and that the latter are also made tense at the same time. There is, in fact, in the membrana tympani at rest no other force capable of holding the radial fibres in a curved position, except the tension of the circular fibres.

In the concussions which sound produces, the pressure of the air acts sometimes upon the convex, sometimes upon the con-

cave surface of the membrana tympani, according as this pressure is alternately greater or less in the meatus than in the cavity of the tympani; in every case the pressure of the air acts perpendicularly upon the membrane, also perpendicularly upon the curve formed by the radial fibres, which curve it at one time increases and at another diminishes. Since the curves formed by the radial fibres of the membrana tympani are only slight, therefore, as will be shown afterward, the mechanical operation is the same as if the pressure of the air were exerted at the end of a very long lever-arm, while the tip of the manubrium represents the end of a very short lever-arm. A relatively great displacement of the surface of the membrana tympani in the same direction as the pressure of the air necessitates a comparatively small displacement of the point of the hammer, and *vice versâ.* Hence, in accordance with the well-known law of virtual velocities, a relatively small amount of pressure of the air will counterbalance a comparatively strong force acting at the handle of the hammer—in other words, it will supply an equivalent force.

In order to understand this, we can limit ourselves to the examination of a single curved radial fibre, which we can suppose changed by the pressure of the air into circular arcs of constant lengths but of differing curves, and hence having differing radii. If, then, l represents the length of the fibre, r the radius of the circle to which the arc belongs, and λ the chord which belongs to the arc l, then is $\frac{1}{2}\ \frac{\lambda}{r}$ the sine of half of the angle at the centre, which belongs to the curve l; therefore,

$$l = 2\ r.\ \text{Arc.}\ \sin\left(\frac{\lambda}{2r}\right)$$

or

$$\lambda = 2r \sin\ \left(\frac{l}{2r}\right)$$

and the difference between the chord and the curve

$$l—\lambda = 2r \left\{\frac{l}{2r}\ -\ \sin\ \left(\frac{l}{2r}\right)\right\}$$

Now, if the curve is very slight—that is, r very large compared with l—then we can suppose the sine of this formula devel-

oped according to the involution of its arc, and limit ourselves to the first of the two divisions of this development, since the divisions become small rapidly.

$$\sin\left(\frac{l}{2r}\right) = \frac{l}{2r} - \frac{1}{6}\cdot\left(\frac{l}{2r}\right)^3.$$

This gives

$$l - \lambda = \frac{1}{24}\,\frac{l^3}{r^2}\ldots\ldots\ldots\ldots\Big\}\ 1.$$

The degree of curvature of the arc, or the distance s of its centre from the centre of the chord, is given by the equation

$$\frac{r-s}{r} = \cos\left(\frac{l}{2r}\right)$$

or

$$s = r\left\{1 - \cos\left(\frac{l}{2r}\right)\right\}$$

If we make here the progressive evolution for the cosine, we have

$$s = \frac{1}{8}\,\frac{l^2}{r}\ldots\ldots\ldots\ldots\Big\}\ 2$$

or, eliminating r from 1 and 2,

$$l - \lambda = \frac{8}{3}\,\frac{s^2}{l}$$

Now, the difference $l-\lambda$ represents the shortening of the chord which is caused by the increase in the curve of the arc, or the extent to which the two ends of the fibre are drawn together. On the other hand, s is the displacement of the middle of the fibre. If, now, s be infinitely small in comparison with l, the length of the fibre, therefore the magnitude $l-\lambda$ in the last formula is an infinitely small magnitude of the second order in comparison with s. The reverse is clear, namely, if we may be permitted to consider the fibre as inextensible, the very small lengthening of the fibre to the amount $l-\lambda$ cannot happen in any other way except that the fibre becomes straightened and its centre experiences the relatively much greater displacement s.

On the other hand, in respect to the estimation of the relation of forces, there is a well-known formula in mechanics, that

the tension t of the fibres, if p represents the pressure upon its unit of length, is represented by the following equation:

$$t = p\,r.$$

The correctness of this formula can be most easily understood when we suppose each fibre everywhere (from end to end) equally curved and perfectly parallel to its similarly curved neighbors, and in this way lengthened to a half circle. Then the forces which draw upon the two ends of the fibre—that is, $2\,t$—must counterbalance the pressure which acts upon the whole diameter of the semi-circle throughout a width equal to that of the fibre—that is, the amount $2\,r.\,p$; and hence the corresponding equation

$$2\,t = 2\,r\,p.$$

Therefore the greater r is, that is to say, the less the curve is under the operation of the pressure of the air, the greater will be the change in the tension produced in the fibre by the pressure of the air.

These changes in the amount of tension of the radial fibres of the membrana tympani are the very ones, however, which the concussions of sound transmit to the handle of the hammer. The amount of tension can increase very considerably under the influence of comparatively slight changes in the pressure of the air, even when the radial fibres of the membrane are stretched out in a very flat curve. It is self-evident that in proportion as the action of this force increases, so the excursions of the handle of the hammer, which can be caused by this force, grow smaller, similarly to that which happens when the intensity of a force is increased by means of a lever.

On the other hand, it is to be remarked that the changes in tension which the pressure of the air induces, can always appear as the increase or diminution of the tension which is maintained by means of the elastic attachments of the membrana tympani and the elasticity of its own radial fibres. A considerable increase in tension, through the pressure of the air from within outward, can only produce a slight effect upon the stirrup, because the articulation of the hammer and anvil yields. Again, on the other hand, the pressure of the air from without can, at the most, only force the handle of the hammer

inward until the radial fibres of the membrana tympani become straight; should the pressure be still greater, then it would curve them again, shorten their chord, and draw the manubrium again outward, provided the circular fibres of the membrana tympani could actually yield so much without breaking, which latter I consider very improbable.

The labyrinth is likewise protected from extremes of pressure, while at the same time the effect of slight variations in pressure can be rendered extremely powerful through the peculiarities of the mechanism already described.

By introducing a manometer into the external meatus, according to the plan proposed by Politzer, it may be shown that the excursion of the parts of the membrana tympani situated in the middle, between the handle of the hammer and the border of attachment, is considerably greater than that of the manubrium itself. In the ordinary anatomical preparations I found it better to fill the meatus entirely with water than to shut up the air contained in the meatus by means of a drop of water in the tube of the manometer. A drop of water so placed resists small displacing forces, since it adheres to the glass tube and does not move when most desirable. If we, however, fill the entire meatus with water, and then introduce the manometer-tube (after having attached to it a suitable plug of sealing-wax) in such a manner that at the same time a certain amount of water will enter it, then the surface of the fluid in the tube will indicate very accurately the displacements of the membrana tympani. As already mentioned, a tube was introduced into the vestibule of the labyrinth in the same preparation, and thus, by forcing in the fluid or withdrawing it, the stirrup and the hammer could be moved.

It has already been stated that in this experiment the excursion of the tip of the handle of the hammer was only $\frac{1}{28}$ mm. The height of the fluid, however, in the manometer varied 1 mm. By calibration with quicksilver, the inside diameter of the tube was found to be 1.37 mm.; the diameters of the membrana tympani were $7\frac{1}{2}$ and 9 mm. From this we can calculate a mean displacement of the membrana tympani of somewhat more than $\frac{1}{9}$ mm.; that is, 3 times as great as the synchronous

movement of the tip of manubrium. Now since the outside border of the membrana tympani is firm, it follows that the middle free parts of the membrane must have experienced a relatively much greater displacement than the amount of the mean displacement given above and therefore more than three times stronger than the tip of the handle of the hammer.

In the foregoing elementary examination of this mechanism we have not taken into consideration the following facts, viz.: that the respective meridional curves of the membrana tympani are closely united; that their distance from one another increases in the direction of the firm border of the membrane; that they are bound together by circular fibres, and that they cannot move without stretching these; in fact, that the naturally curved form of the membrana tympani cannot exist without its circular fibres being extended and made tense by every force which draws the handle of the hammer inward.

The form of the membrana tympani being so irregular, a perfect analysis of the mechanical action of the parts cannot be given. It would be necessary first to know the tension and the measure of the elasticity of the circular fibres. We can, however, make a mathematical representation which would better correspond to the actual relation of the parts, if, instead of the real membrana tympani, we imagine an ideal one, which is conical in the centre, but toward the periphery is curved and symmetrical, and represents therefore a surface of rotation. The radial fibres, which follow the direction of the meridians of such a surface, can be considered as incapable of extension: the circular fibres, however, must possess a certain degree of elasticity in order to remain always tense. In the appendix, I have developed the theoretical question regarding the mechanical workings of such a membrane, and the most advantageous form to give it. It will be sufficient here to remark that the pressure of the air will produce the strongest effect upon a slightly curved membrane, when it has, by means of its own elasticity, taken the form which the pressure of the air tends to give it. This form is one where with unchanged length of the radial fibres and unchanged position of its centre, the volume of its concave side, that is, the volume of the cavity of the

drum becomes a maximum, and that on its convex side is reduced to a minimum. If the membrane had not originally possessed such a form, still the pressure of the air would have produced such a result by changing the tension of the circular fibres, before it could have excited its entire force upon the centre.

The form here required of a circular membrane, can be calculated—the transverse section of such an one, in some degree corresponding to the relation of the membrana tympani, is given in Fig. 10. This form will be seen to coincide well with the

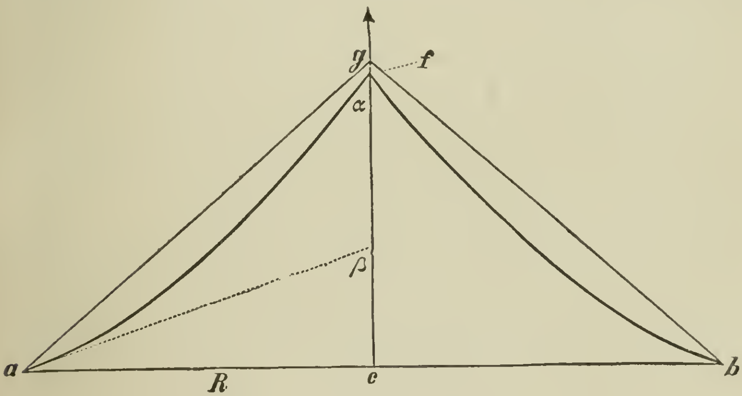

Fig. 10.

relatively free lower portion of the membrana tympani.

Let a represent the angle which the tangent of the membrane drawn through its apex (umbo) in the meridian-plane makes with the axis; β, that which the corresponding tangent of a point in the periphery of the membrane makes with the axis; R, the radius of the circle at the periphery; f, the pressure of the air; then will k be the force which must be applied at the centre of the membrane to counterbalance the pressure of the air:—

$$k = \frac{p\pi R^2 \cos a}{\cos a - \cos \beta}.$$

In this equation we see once more that the smaller the difference between the two angles a and β, that is, the shallower the curve made by the the tense radial fibres of the membrane, the

stronger will be the force. Further, the force increases as the *cos* α, if the angles α and β become smaller, while the difference *cos* α — *cos* β remains the same, that is, if the apex of the membrane be drawn in more strongly.

Thus far the acoustic action of such curved membranes has not yet been practically studied. It may be proper to state here, that in the Tunis Café at the Paris industrial exhibition, I saw a curved piece of leather employed as a sounding board in an Arabian stringed instrument. A membrane similar to the membrana tympani can be made by stretching a wet piece of a pig's bladder over the upper end of a glass cylinder: the cylinder should be placed in an upright position, then place a rod loaded with metal perpendicularly to the centre of the membrane, so that its lower end presses the centre of the bladder downward. In this position the bladder must be allowed to dry. It will then retain permanently a form similar to that of the membrana tympani, with its retracted navel and its curved meridian lines whose convexity looks outward.

In order to test the acoustic action of such a membrane under relations similar to those in which the membrana tympani is placed, I fastened the cylinder, whose inside diameter amounts to 44 mm., to a strong wooden board (A, Fig. 11.) (In the figure the cylinder is situated between *e* and *f*, and has been

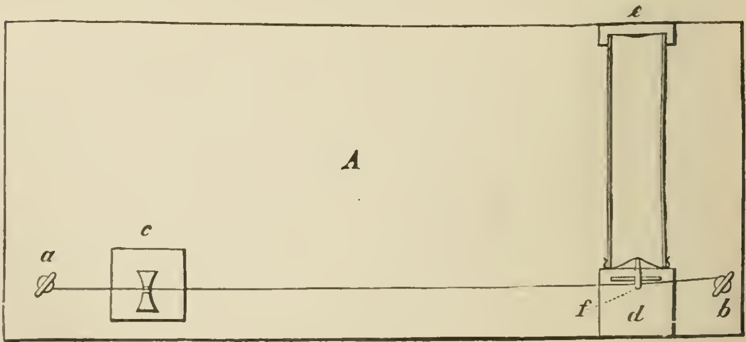

Fig. 11.

represented as cut in two. The cylinder was then tied to the board, its open end at *e*, having first been supported by a piece of wood, properly cut out to receive it, by which arrangement

any sliding movement in the direction of *e* was prevented. A small, light wooden rod was then placed upon the retracted centre of the membrane and this served as a bridge for a string stretched between two pegs at *a* and *b*. At *c* the string runs over a bridge placed on the centre of a block of lead, beyond which, of course, the string cannot vibrate. Another block of lead was placed at *d*, and between it and the rod *f*, a thin board like the bridge of a violin was inserted parallel to the string. This board supported the rod, but offered no obstruction to the shocks which the rod received in the direction of its own length, from the string.

The blocks of lead serve to weaken the transmission of the vibration of the string to the board and so through it to the atmosphere, so that when we draw a violin bow across the string and at the same time lift up the rod *f* from the curved membrane, or grasp the string with the fingers at a point near *f*, between this point and *c* the resulting sound will be very much muffled. As soon, however, as the vibrations of the string can pass through the rod to the curved membrane, the latter, notwithstanding its suddeness, gives forth almost as powerful a tone as the violin itself. The string can easily be shortened by holding it between two fingers of the left hand, while with the right hand we draw the bow, which should be placed near the fingers of the left hand. It is evident then, that this powerful resonance extends over the greater portion of the scale, and in the case of the high tones in the middle of the octave c_4 . . c_5 it reaches such an intensity that they can hardly be endured.

This process resembles that which takes place in the membrana tympani, in so far as the curved membrane serves to transmit the vibrations from the air to a solid body of moderate weight and relatively small amplitude of vibration, as for instance the labyrinth-water on the one hand, and the ends of the strings on the other. If, however, sound be easily transmitted from the string to the air, then the reverse must also easily take place, according to the general law of reciprocal forces for oscillations of sound in perfectly elastic bodies.[1]

[1] The law for masses of air inclosed in firm walls is stated and proven in an essay published by me in the "Journal für reine und angewandte Mathematik." Bd. LVII, p. 29. Equation 92. The title is, "Theorie der Luftschwingungen in Röhren mit offenen Enden."

The proof of this can easily be found by experiment on the above-mentioned apparatus. If we place small paper riders (or thin fibres of wood) upon the string and sing the tone which belongs to it over the mouth of the tube, the paper figures will begin to dance. The tone of a tuning fork placed upon a sounding board will also cause a string of the same pitch with it to resound, and the paper figures will dance. The same effect will be produced when the tuning fork is held at a distance of several feet from the string. The pitch, moreover, of the glass tube exerts an influence similar to that which takes place when the ear is armed with a resonator. If we make the string of such a length that its fundamental tone agrees with the tone of the tube, then the tone of the string will be particularly full and powerful.

§ 8.

MATHEMATICAL APPENDIX,

having particular reference to the mechanism of curved membranes.

In the following we presuppose a membrane of circular form, having inextensible meridian lines, and tense elastic fibres; we assume also that p the pressure of the air acts upon one of the surfaces of this membrane, and that, on the other hand, there is present a force g acting upon its centre in the direction of symmetry.

Then in order to understand, as shown in the preceding section, how the pressure of the air produces the strongest resultants in the centre of a feebly curved membrane (provided the membrane, through the action of its elastic circular fibres, has assumed the same form which the pressure of the air would give to it, were the elastic tension of the circular fibres wanting), let us take into consideration the following propositions.

In Fig. 10, page 59, let ab be a diameter, c the middle point of the firm border of the membrane, and f the centre of the membrane, which centre the force fg draws in the direction of the axis. The membrane may have assumed the form indicated by the curved line merely through the influence of the tension of its elastic circular fibres and be thus in permanent equilibrium.

We will next presuppose that the air presses equally upon both sides of the membrane.

Now it is a well known general law in mechanics, that in every case, where the law of the conservation of forces comes into play, permanent equilibrium takes place only when, among all the positions which the movable system can assume, the condition of equilibrium is that one in which the measure of internal and external forces acting upon it is a maximum.

This law is applicable to the membrane in question and it follows that in its condition of equilibrium the total amount of force exerted by the contraction of the elastic circular fibres must be a maximum in comparison with that which may be exerted in any of the other forms into which the membrane could pass while the position of the point f remains unchanged.

Should then any other force whatever give the membrane another form, while the position of the point h remains unchanged, the work accomplished by this force must necessarily be of a positive character, since the quantity of the power of the tension exerted by the membrane must be increased by this transition.

The same would hold true if the membrane were brought into the position afb, not by means of its elasticity but through the pressure of the more condensed air above it, exerting a force fg upon its centre f. In this case the membrane must necessarily assume such a form that the force, exerted by the expansion of the more condensed air above, would be a maximum. This latter would however take place, if the volume of the air contained above the membrane and the prolonged plane ab became a maximum. It follows again, that if another force were employed in order to change the form of the membrane in any way whatever, the volume of the more condensed air above would necessarily become less and the additional force would have to produce positive results.

Now, if the form afb, produced by the elastic force, be exactly the same as that which the pressure of the air produces and if the former counterbalances the force g and the latter the force γ, then will the membrane without change of form be in equilibrium, through the simultaneous action of the elastic circular fibres and the pressure of the air, and will counterbalance the

force g and γ which acts at the point f. If the position which the membrane assumes, when in a position of equilibrium through the action of elastic forces (the centre of the membrane being at f'), differs from that which the pressure of the air produces (the position of the centre being still the same), then the membrane under the united influence of both forces will come to a state of rest in a position varying from either of the other two positions. In this position, neither the elastic forces nor the pressure of the air will have exerted the maximum of their power, that is, what they are capable of doing when the centre is at f.

Taking then, as a starting-point, that form which the membrane receives, when the force g is infinitely great and when, as a result, the radial fibres must be extended in a straight line, and then supposing the force g to be gradually diminished until the centre of the membrane has advanced to the point f, we shall find that the membrane exerts a force which increases in value from zero to a value of G, and that this value is dependent upon the position of the point f.

Let then G_0 represent the force exerted in this case, when the elasticity alone acts, G_1 when the pressure of the air alone acts, and G_2 when both forces act simultaneously; then $G_2 < G_0 + G_1$, except in the case where the elasticity and the pressure of the air give the same form to the membrane.

Starting from the position where these quantities are equal to zero, then, if the length $g\ f$ be represented by h, during a first period must

$$\frac{d\ G_2}{d\ h} < \frac{d\ G_0}{d\ h} + \frac{d\ G_1}{d\ h},$$

because otherwise from the outset the following would have been true:

$$G_2 \leqq G_0 + G_1.$$

Now, however, the above differential quotients equal the resulting forces which tend to draw the centre of the membrane toward c.

The force with which the elasticity of the membrane (taken alone) acts, is represented by g:

$$g = \frac{d\ G_0}{d\ h}$$

By γ is represented the force with which the pressure of the air, taken alone, acts:

$$\gamma = \frac{d\, G_1}{d\, h}$$

and the force, with which the pressure of the air and the elasticity together act, we will indicate by $g + \gamma_0$:

$$g + \gamma_0 = \frac{d\, G_0}{d\, h}$$

It follows from the above equation that in the smaller curves of the membrane

$$g + \gamma_0 < g + \gamma,$$

or

$$\gamma_0 < \gamma,$$

provided the form of the membrane, in which the condition of equilibrium exists, is not the same for the simple force of elasticity and for the pressure of the air; *Q. E. D.*

To determine the form of a membrane made tense by the pressure of the air alone, and containing inextensible radial fibres.

Let z be a given portion of the axis of the membrane, and r the radius of the circle in which a plane, passing through a variable point near the end of z perpendicularly to the axis, cuts the membrane. The volume which lies between two such planes, corresponding to the values z and $z + d\, z$, whose difference is infinitely small, is therefore

$$\pi\, r^2\, d\, z$$

The entire volume v, between the membrane and the plane which passes through its circle of attachment, is therefore

$$v = \int_o^a \pi\, r^2\, d\, z$$

if for the centre of the membrane $z = o$ and for the circumference $z = a$.

Let p represent the excess of the pressure of the air, upon the upper surface of the membrane, over the pressure of the air on the under surface, and G the effects of a force acting upon the centre of the membrane and in a direction parallel to its axis, then the combined effect of this force and the pressure of the air is equal to

$$G - p\, v.$$

The conditions which permit of equilibrium are that this quantity should be a maximum while the length of the radial fibres remains the same; the element of this length has been given in the equation

$$ds^2 = dr^2 + dz^2.$$

Considering then r as an independent *variable*, then must

$$G - p\,\pi \int_o^R r^2 \frac{dz}{dr} dr = \text{Max.}$$

or, according to the principles of differential calculus, if we differentiate z:

$$\frac{dG}{dz}\,\delta z - \pi\,p \int_o^R \left(r^2 \frac{d\delta z}{dr} - \frac{\lambda \frac{dz}{dr} \frac{d\delta z}{dr}}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}} \right) dr = 0.$$

Partially integrated we have the following result, if we make $\delta\,z$, at the periphery of the membrane, equal 0, and in its centre equal $\delta\,z_0$:

$$\left\{ \frac{d\,G}{dz} + \pi\,p\,\lambda \frac{\frac{d\,z}{dr}}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}} \right\} \delta z_0$$

$$+ \pi\,p \int \delta z \frac{d}{dr} \left(r^2 - \lambda \frac{\frac{d\,z}{d\,r}}{\sqrt{1 + \left(\frac{d\,z}{d\,r}\right)^2}} \right) dr = 0$$

Since then dz_0 and dz are absolute quantities independent of each other, it follows that those magnitudes which are multiplied by them equal zero; hence,

1. For the centre of the membrane:

$$\frac{dG}{dz} + \pi\,p\,\lambda \frac{\frac{dz}{dr,}}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}} = 0\,;$$

2. For its surface,

$$r^2 - \lambda \frac{\frac{dz}{dr}}{\sqrt{1 + \left(\frac{dz}{dr}\right)^2}} = C,$$

where C is understood to represent a constant. In the central point of the membrane $r = o$, and $\frac{dz}{dr} = cot.\ a$, a representing the angle so designated in Fig. 10. For this point therefore equation No. 2 becomes reduced to $C = -\lambda\, cos\ a$ and the equation No. 1 gives for the same point

$$\frac{d\,G}{d\,z} + \pi\, p\, \lambda\, cos\ a = 0.$$

When we, on the other hand, represent the amount r at the border of the membrane by R, and let $\frac{dz}{dr} = tang\ \beta$, as in Fig. 10, then according to equation No. 2

$$R^2 - \lambda\, cos\ \beta = C = -\lambda\, cos\ a$$

consequently

$$R^2 = \lambda\, (cos\ \beta - cos\ a)$$

and the force g

$$g = \frac{dG}{dz} = -\frac{\pi\, p\, R^2\, cos\ a}{cos\ \beta - cos\ a}$$

as given in the previous section.

Again, from equation No. 2 follows:

$$(r^2 + \lambda\, cos\ a)^2 \left[1 + \left(\frac{dz}{dr}\right)^2\right] = \lambda^2 \left(\frac{dz}{dr}\right)^2$$

or

$$\frac{r^2 + \lambda\, cos\ a}{\sqrt{\lambda^2 - (r^2 + \lambda\, cos\ a)^2}} = \frac{dz}{dr}$$

This is an elliptical integral which we restore to the normal form when we make

$$r = \sqrt{2\, \lambda\, sin\, \frac{a}{2} . cos\, \omega}$$

$$dr = -\sqrt{2\, \lambda\, sin \frac{a}{2} . sin\, \omega\, d\, \omega}$$

then will

$$dz = -\sqrt{\frac{\lambda}{2}}\,\frac{1 - 2\sin^2\frac{a}{2}\sin^2\omega}{\sqrt{1 - \sin^2\frac{a}{2}\sin^2\omega}}\,d\omega$$

Or, if we follow Legendre,

$$F\omega = \int_o^\omega \frac{d\omega}{\sqrt{1 - \chi^2 \sin^2\omega}}$$

$$E\omega = \int_o^\omega \sqrt{1 - \chi^2 \sin^2\omega}\, d\omega$$

and place

$$\chi^2 = \sin^2\frac{a}{2}$$

then is

$$z = \sqrt{\frac{\lambda}{2}}\left\{2\,E\omega - F\omega\right\} + Const.$$

$$r = 2\sqrt{\frac{\lambda}{2}}\cdot\chi\cos\omega$$

At the same time we easily find the length of the arc of the radial fibres—

$$s = \sqrt{\frac{\lambda}{2}}\,F\omega$$

By means of Legendre's tables, which give the values of $E\omega$ and $F\omega$ for all values of $\frac{a}{2}$ and ω, which correspond to whole degrees, we can construct the form of this curve in the easiest possible manner. For arbitrary values of a and ω the values of $E\omega$ and $F\omega$ can be computed according to well-known methods.

Fig. 12 shows a perfect curve of this kind, drawn from one axis-point to the other, in which the value $180° - 40° = 140°$ is given to the angle a, corresponding to the form of the membrana tympani. The axis-point may represent the centre of the membrane. Each point of the arms of the curve extending from a could correspond to the circumference of the membrane, as far

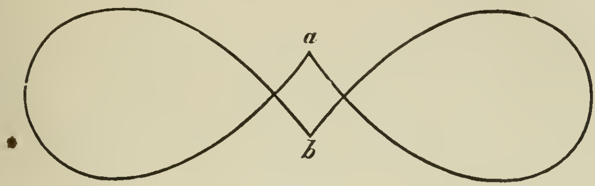

Fig. 12.

as that point where the curve, descending in the direction of *b*, meets and crosses itself again. The membrana tympani itself corresponds only to a small part of this curve.

For the present, I shall defer the special description and discussion of my experiments (referred to in foot-note of page 8) on "resonance-tones" in the living ear, because I hope to obtain better means of producing deep and simple tones than I have had thus far, and in order that the experiments may be better performed.